PERCEIVED SURFACE COLOR IN BINOCULARLY-
VIEWED SCENES WITH TWO LIGHT SOURCES
DIFFERING IN CHROMATICITY

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ABSTRACT

We examine the relationship between the perceived orientation and the perceived color of matte surfaces in rendered three-dimensional scenes illuminated by a blue diffuse light and a yellow punctate light. On each trial, observers first adjusted the color of a matte test patch, placed near the center of the scene, until it appeared achromatic, and then estimated its orientation by adjusting a monocular gradient probe. The orientation of the test patch was varied from trial to trial, effectively varying the chromaticity of the light mixture from the two light sources that would be absorbed and re-emitted by a neutral test patch. We found that observers’ perception of the orientation of the test patch was nearly veridical and that they substantially discounted the effect of changing orientation in estimating perceived color. Observers were nearly veridical in estimating the spatial distribution of light sources but erred substantially in judging their chromaticities.
INTRODUCTION

In a field, on an ordinary sunny day, the light impinging on each object is a mixture of direct sunlight, light from sky and clouds, and light absorbed and re-emitted by other objects in the scene. Even when the contribution from other objects is neglected, the light absorbed and re-emitted by any surface is composite, a mixture that changes with each passing cloud. Our goal is to examine how human observers perceive surface colors in scenes with composite lighting. In particular, we will test whether they correctly discount surface orientation in estimating the surface color of matte surface patches in simulated scenes illuminated by a yellow punctate source and a blue diffuse source.

In Figure 1, we illustrate how light is absorbed and re-emitted by a Lambertian (matte) surface illuminated by a punctate source and a diffuse source. The intensity of the punctate source at a wavelength $\lambda$ is denoted by $E_p(\lambda)$ and the intensity of the diffuse light by $E_d(\lambda)$. The angle between the punctate light direction and the surface normal $n$ is denoted by $\theta$, and the angle between the surface normal and the direction to the viewer is denoted by $\nu$. In the Lambertian model, the intensity of emitted light does not depend on the direction to the viewer, so long as the viewer and the light source are on the same side of the surface. When this condition is satisfied, the intensity of the light reflected from an achromatic Lambertian surface at any wavelength $\lambda$ is given by

$$E(\lambda) = \alpha \left( E_p(\lambda) \cos \theta + E_d(\lambda) \right),$$

(1)

where $\alpha$ is the wavelength independent albedo (reflectance) of the achromatic surface.
Previous Research

A number of researchers have investigated how the spatial arrangement of lights and surfaces in a scene affects the perception of lightness or color of a particular matte surface in the scene.

Gilchrist (1977, 1980; Gilchrist et al., 1999) examined scenes where the intensity of illumination varied with depth and found that perceived depth affected the perceived lightness of achromatic surfaces. The experimental setup was composed of two rooms which differed in illumination, and were connected by a doorway. Observers viewed this construction through a pin hole. In the two conditions of his experiment the target patch, whose lightness was to be matched, would be perceived to be located either as coplanar with a brightly illuminated far wall or a dimly illuminated near wall. The actual position and brightness of the test patch were not altered. The test patch was perceived to be white in the near wall condition and almost black in the far wall condition. Gilchrist argued that perceived lightness depends on the relationship between the target and regions with which it is seen as coplanar (the coplanar ratio hypothesis).

Boyaci et al. (in press) set out to determine whether perceived orientation affects perceived lightness. Their stimuli were binocularly viewed, computer-rendered scenes illuminated by a neutral, punctate light source and a neutral diffuse light source. Observers first estimated the orientation of an achromatic test patch with Lambertian reflectance properties and then matched its perceived albedo (‘lightness’) to a reference scale. Their results clearly showed that observers systematically discounted the perceived orientation of a surface when estimating its albedo. In fact, observers’ performances closely matched the predictions of the Lambertian model.
How dramatically the perception of 3D shape can influence perceived surface reflectance was demonstrated by Bloj et al. (1999). They used a chromatic version of the Mach card; one side of the card was painted magenta and the other white, the magenta side casting a pinkish gradient on the white areas. With binocular disparity as the only cue to shape, observers viewed the card (1) in its actual concave shape and (2) through a pseudoscope which reversed the disparities in left and right eye so that the card appeared to be convex. Bloj and colleagues found significant evidence that observers incorporate information about the shape of the object into their estimates of surface color: the color of the white side of the card was judged by observers to be more pinkish in the apparently convex condition than in the actual concave condition.

Bloj and colleagues studied the effect of 3D shape on color perception for only two viewing conditions. What happens for different angles between the mutually illuminated surfaces? According to the physics of light the strength of the mutual illumination will depend on the angle between the emitting and receiving surface. Doerschner et al. (under review) showed that observers systematically discount the angle between a brightly colored surface and an adjacent light gray surface, when setting the color of the latter to be achromatic.

Yang and Shevell (2003) investigated surface color appearance in simulated scenes illuminated by two punctate light sources differing in chromaticity. The scenes were rendered and presented binocularly, and each consisted of two side-by-side rooms separated by an opaque partition oriented along the line of sight. The back wall of each room was covered with lozenge-shaped specular objects. The light sources were placed so that each primarily illuminated one of the rooms and only secondarily the other (thus
the primary light source for each room was the secondary for the other). Yang & Shevell varied the contribution of the secondary light source by varying the height of the partition.

Color constancy was greatest when each room was illuminated exclusively by its primary light source and decreased with increasing admixture of the secondary. In these scenes, each light source created a distinct specular highlight on each of the specular lozenges that it illuminated. Yang & Shevell perturbed the chromaticity of these highlights to show that they were effective as cues to the chromaticity of the light reaching each of the room, confirming earlier results (Yang & Maloney, 2001; Yang & Shevell, 2002; Maloney & Yang, 2003).

The results of Yang & Shevell suggest that the human visual system can discount the relative contributions of two light sources in a scene, at least when there is sufficient information in the scene to permit estimation of the chromaticities and spatial distribution of the light sources. In the experiment we describe next, we will provide considerable visual information about the chromaticity of the light sources and the direction to the punctate light source.
EXPERIMENT

Introduction

In this experiment we asked observers to carry out two tasks on each trial. They first set a Lambertian test patch to be achromatic (Helson & Michels, 1948) and then set a gradient probe to match its orientation. The test patch was embedded in a scene illuminated by a mixture of a blue diffuse light source and a yellow punctate light source. We varied the orientation of the test patch with respect to the punctate light source from trial to trial, thereby varying the chromaticity of the composite illuminant striking the test patch. We sought to determine whether observers compensated for changes in orientation (and illuminant chromaticity) in their achromatic settings.

Methods

**Stimuli.** The stimuli were computer-rendered three-dimensional complex scenes composed of simple objects with different shapes (such as spheres and boxes), and with various reflectance properties (such as shiny, matte and transparent). Each scene contained a matte test patch at the center. The scenes were rendered with the **Radiance** software package (Larson and Shakespeare, 1996). Each scene was rendered twice, from slightly different viewpoints corresponding to the positions of the observer’s eyes. A stereo image pair for a typical scene is shown in Figure 2. The other objects in the scene were varied randomly from trial to trial and were included as possible cues to the spatial distribution and chromaticities of the light sources (see Yang & Maloney, 2001).

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Figure 2: Sample Stimulus
**Apparatus.** The experimental apparatus was a Wheatstone stereoscope (Figure 3). The left and right images of each stereo pair were presented to the corresponding eye of the observer on two 21" Sony Trinitron Multiscan GDM-F500 monitors placed to the left and right of the observer. The screens on these monitors are close to physically flat, with less than 1 \( \text{mm} \) of deviation across the surface of each monitor.

The stereoscope was contained in a box 124 \( \text{cm} \) on a side. The front face of the box was removed, and the observer sat there in a chin/head rest. Two small mirrors were placed directly in front of the observer's eyes. These mirrors reflected the images displayed on the left and right monitor to the corresponding eye of the observer. The interior of the box was coated with black flocked paper (Edmund Scientific) to absorb stray light. Only the stimuli on the screens of the monitors were visible to the observer. The casings of the monitors and any other features of the room were hidden behind the non-reflective walls of the enclosing box. Additional light baffles were placed near the observer’s face to prevent light from the screens reaching the observer’s eyes directly. The optical distance from each of the observer’s eyes to the corresponding computer screen was 70 \( \text{cm} \). To minimize any conflict between binocular disparity and accommodation depth cues, the center of the test patch was rendered to be exactly 70 \( \text{cm} \) in front of the observer. The monocular fields of view were 55 × 55 \( \text{deg} \). The observer’s eyes were approximately at the same height as the center of the scene being viewed which was also the position of the center of the test patch.

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**Figure 3: Apparatus**
**Spatial coordinate system and spatial arrangement.** We used a spherical coordinate system \((\psi, \varphi, r)\) to specify a simulated scene (Figure 4). This coordinate system has the origin at the center of the test patch. The spherical coordinate system \((\psi, \varphi, r)\) and the Cartesian coordinate system underlying it are explained in the figure legend.

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**Figure 4: Spatial coordinate system**

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**Color coordinate system.** We will describe all light sources and the light radiating from surfaces as weighted mixtures of three abstract primary lights referred to as red, green and blue (R, G, and B). For convenience, the spectra of these lights coincide with those of the corresponding guns of the monitors, and the three primaries can be thought of as linearized versions of the guns, for that is what they are. We measure the intensities of these three primaries in arbitrary units proportional to their luminance, chosen so that a mixture of the three lights with equal intensities appears roughly achromatic to most observers. We denote the intensities by \(E^R\), \(E^G\), and \(E^B\), respectively, and refer to the 3-vector \((E^R, E^G, E^B)\) that describes the light at a particular location on the monitor as an RGB code. In making an achromatic setting, the observer in effect selects the RGB code for the test patch that makes it appear to be an achromatic surface, as described in more detail below. We report the u’v’ chromaticities (Wyszecki & Stiles, 1982, p. 165) of the guns (and, therefore, of the primaries) in Calibration below.

**Realism and rendering.** Typical rendering packages used in computer graphics represent spectral information about surfaces and light sources by three dimensional vectors, often referred to as RGB codes. When light with descriptor \((E^R, E^G, E^B)\) strikes
a Lambertian surface with descriptor \((S^R, S^G, S^B)\), the light emitted from the surface is assigned the descriptor \((E^R \times S^R, E^G \times S^G, E^B \times S^B)\), scaled by a factor that takes into account the orientation of the surface with respect to the light (see the discussion leading up to Equation 1). Yang & Maloney (2001; Maloney, 1999) point out that this rendering interpretation (‘the RGB heuristic’ in Maloney, 1999) does not always lead to accurate simulation of light-surface interactions.

However, the scenes that we use are designed to avoid the limitations of typical rendering packages. First, we define an achromatic Lambertian surface to be one that multiples the chromaticity of the light that it absorbs and re-emits by a constant factor that depends upon the albedo of the surface and the direction from which the light arrives. If we assign this neutral surface the RGB-code \((\alpha, \alpha, \alpha)\) then typical rendering packages will simulate light-surface interaction correctly for an achromatic Lambertian surface. So long as our chromatic lights interact with only neutral surfaces, the resulting RGB codes assigned to the light re-emitted will be accurate. There are other surfaces in our scenes that are rendered, but the RGB codes of these surfaces are assigned at random and change from trial to trial. Consequently, errors in rendering, due to using the RGB heuristic, are of no consequence. The intended random color assigned to a surface is just replaced by a different random color.

We can derive three equations that break Equation 1 into three RGB-chromaticity components. For \(B\), we have,

\[
E^B(\theta) = E_p^B \cos \theta + E_D^B
\]  

(2)
and there are two analogous equations for $R$ and $G$, respectively. When the test patch is achromatic, each of the components of the RGB-code of the light emitted by the test patch is the same weighted mixture of the corresponding components of the two light sources.

**Calibration.** Look-up tables were used to correct the nonlinearities in the gun responses and to equalize the display values on the two monitors. The tables were prepared after direct measurements of the luminance values of each gun on each monitor with a Pritchard PR-650 spectrometer. The maximum total luminance achievable on either screen was $114\, cd/m^2$. To test the linear additivity for a monitor, first we measured the isolated spectrum of each gun alone, set to about half of its maximum intensity. Then we measured the spectra of each pair of guns simultaneously set to half of their maximum intensities and compared it to the sum of the isolated spectra for each gun in the pair. Last, we measured the spectrum with all three guns set to half of their maximum intensity and compared it to the sum of the isolated spectra for all three guns. We plot the results of this last test in Fig. 5 for both monitors. The red, green and blue solid lines are the isolated spectra, the gray solid line is the sum of the three isolated spectra, and the black dashed line is the measured spectra when all three guns were simultaneously set to half of their maximum intensities. The curves agree to within 7% or better at each point in the spectrum, for both monitors. The test of additivity for pairs of guns also agreed within 7% or less. The $u'v'$ chromaticity coordinates (Wyszecki & Stiles, 1982, p. 165) for the three primaries are: Red (.409,.519), Green (.117,.565) and Blue (.157,.196) for the left monitor and Red (.430,.528), Green (.115,.564) and Blue
(.160,.189) for the right monitor. The u’v’ chromaticity coordinates for the mixture of all three guns at half intensity was (.176,.460) for the left monitor and (.172,.455) for the right monitor.

Figure 5: Gun additivity

Light sources. A yellow punctate light source and a mostly blue diffuse light source illuminated the scenes. The RGB chromaticity coordinates of the punctate light source are denoted \((E_p^R, E_p^G, E_p^B)\) and that of the diffuse light source are denoted \((E_D^R, E_D^G, E_D^B)\). For convenience, let \(E_p = E_p^R + E_p^G + E_p^B\) and \(E_D = E_D^R + E_D^G + E_D^B\). To specify the chromaticities of the two light sources and their relative strengths we define the following set of parameters: \(\pi^B = E_p^B / E_p\), the blue balance of the punctate source, \(\delta^B = E_D^B / E_D\), the blue balance of the diffuse source, and \(\Delta = E_D / E_p\), the diffuse-punctate ratio. The values used in rendering were \(\pi^B = 0\), \(\delta^B = 0.66\) and \(\Delta = 0.37\). In other words, the punctate source had no blue component \((\pi^B = 0)\), the diffuse source was mostly blue \((\delta^B = 0.66)\), and the ratio of the intensity of the punctate source to the intensity of the diffuse source was 0.37 \((\Delta = 0.37)\). The intensities of the green and red components of the yellow punctate source were equal, as were those of the diffuse source. The punctate source was always behind and above the observer, and either to his RIGHT or to his LEFT at \((\psi_p, \varphi_p, r_p) = (\pm 15^\circ, 30^\circ, 670 \text{ cm})\) \((\psi_p = +15^\circ \text{ for RIGHT, } \psi_p = -15^\circ \text{ for LEFT}; \text{ Figure 4})\). The position of the punctate source was varied only from session to session, but in a single session the position of the punctate source was kept constant. The
punctate source was sufficiently far from the test patch so as to treat its light rays collimated. The vector \( \mathbf{p} = (\cos \varphi_p \sin \psi_p, \sin \varphi_p, \cos \varphi_p \cos \psi_p) \) is a unit vector pointing from the test patch toward the punctate light source (Figure 4).

**Test patch.** Each scene contained a test patch at the center, which was originally rendered as an achromatic Lambertian surface with an albedo of 0.55 (the observer never saw this patch and we used it only to verify that the output of the Radiance program agreed with the predictions of Eqs. 2 and 3. The part of the left and the right images corresponding to this patch were replaced by a uniform test patch before the images were shown to the observer). The initial chromaticity of the substituted test patch was chosen at random before each trial. The test patch could be displayed with either a rotation in only the \( \psi \) direction (\( \psi - rotation \)) or in only the \( \varphi \) direction (\( \varphi - rotation \)). The test patch measured 4.8 cm by 3.6 cm; its center of gravity was always 70 cm away from the observer along the observer’s line of sight. The orientation of the test patch was specified by \((\psi_T, \varphi_T)\), and its surface normal was \( \mathbf{n} = (\cos \varphi_T \sin \psi_T, \sin \varphi_T, \cos \varphi_T \cos \psi_T) \). After a \( \psi - rotation \) (\( \varphi_T = 0 \)), \( \psi_T \) could take any of the values \( \{-60^\circ, -45^\circ, -15^\circ, 15^\circ, 45^\circ\} \) when the punctate source was on the LEFT, and any of the values \( \{-45^\circ, -15^\circ, 15^\circ, 45^\circ, 60^\circ\} \) when the punctate source was on the RIGHT. After a \( \varphi - rotation \) (\( \psi_T = 0 \)), \( \varphi_T \) could take any of the values \( \{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ\} \). Figure 6 shows a schematic drawing of the two kinds of rotations. The test patch floated in space in the middle of the scene. It was closer to the observer than all other objects and sufficiently high above the floor so that we could eliminate possible mutual illumination effects.

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**Figure 6: Orientations**
Angle of incidence. The cosine of the angle between the surface normal of the test patch and the direction to the punctate source is found by employing the law of cosines

\[
\cos \theta = n \cdot p = \cos \phi_r \cos \psi_T \sin \psi_p \\
+ \sin \phi_r \sin \phi_p + \cos \phi_r \cos \phi_p \cos \psi_T \cos \psi_p, 
\]

or

\[
\cos \theta = \begin{cases} 
\cos \phi_p \cos (\psi_p - \psi_T) & \text{for } \psi - \text{rotation} \\
\sin \phi_r \sin \phi_p + \cos \phi_r \cos \phi_p \cos \psi_T & \text{for } \phi - \text{rotation}
\end{cases}, 
\]

because \( \phi_r = 0 \) for a \( \psi - \text{rotation} \) and \( \psi_T = 0 \) for a \( \phi - \text{rotation} \).

Tasks. The observer carried out two tasks on each trial.

Achromatic Setting Task. The observer first adjusted the chromaticity of the test patch such that it looked achromatic. He or she did this by varying the blue-yellow and red-green balances without altering the total intensity of the test patch (Figure 7a). The observer used the arrow keys on the keyboard to perform this task. Pressing the up arrow key increased the blue content while decreasing the yellow (red + green) content by the same amount, the down arrow had the opposite effect. Pressing the right arrow key increased the red content while decreasing the green content, the left arrow increased the green content and decreased the red content by the same amount. The key variables here were the red, green and blue intensities that characterized the observer’s final choice of setting: \( \hat{E}^r \), \( \hat{E}^g \) and \( \hat{E}^b \).

Orientation task. The second task was to estimate the orientation of the test patch by adjusting a stick-and-circle gradient probe superimposed at the center of the test patch
The orientation of the probe was controlled by moving a computer mouse. The probe was presented monocularly to the right eye and the observer had only one degree of freedom on each trial: if it was a $\varphi$–rotation trial, the probe could rotate only in the $\varphi$ direction, if it was a $\psi$–rotation it could rotate only in the $\psi$ direction. Observers reported no difficulty with setting the probe and were unaware that it was visible in only the right eye. Once the observer was satisfied with the setting, he or she clicked the left button on the computer mouse to finalize the task.

**Software.** The experimental software was written by us in the C language. We used the X Window System, Version 11R6 (Scheifler & Gettys, 1996) running under Red Hat Linux 6.1 for graphical display. The computer was a Dell 410 Workstation with a Matrox G450 dual head graphics card and a special purpose graphics driver from Xi Graphics that permitted a single computer to control both monitors. We use the open source physics-based rendering package Radiance (Larson & Shakespeare, 1996) to render the left and right images that comprised the stereo pair for a given virtual scene. The output of the rendering described above was a stereo image pair with floating point RGB triplets for each pixel. These triplets were interpreted as the relative red, green, and blue intensity values that would arrive at points in the retinas if the observer’s eyes were at the viewpoints selected in the virtual scene. We translated the output relative intensity values to 24-bit graphics codes, correcting for nonlinearities in the monitors’ responses as described above.
**Procedure.** The observers repeated each of the 20 conditions (10 test patch orientations, 2 punctate source positions) of the experiment 20 times for a total of 400 trials. The experiment was split into four sessions, each with 100 trials. In a single session the position of the punctate source (LEFT or RIGHT) was kept constant. The order of presentation was randomized. The observers were allowed to perform a few trials before the actual experiment started, until they were completely comfortable with both tasks. The experiment was paced by the observer. Usually observers completed different sessions on different days and each session took less than an hour.

**Observers.** Four observers completed the experiment. All were experienced psychophysical observers who were unaware of the purpose of the experiment. One other observer was excused after the first session. She had difficulty doing the task and spent more than 3 hours to finish a single session (which usually took other observers under an hour).

**Instructions to the observer.** For the color task, we asked the observer to adjust the color of the test patch such that it looked as if it were cut out of an achromatic or gray piece of paper. For the orientation task the observers were simply instructed to move the mouse until the probe’s circles were in the plane of the test patch and the stick was perpendicular to it.

**Geometric chromaticity balance function.** In order to quantify the observers’ perception and compare it with the model predictions, we define

\[
\Lambda^B(\theta) = \frac{E^B(\theta)}{E(\theta)} = \frac{E_p^B \cos \theta + E_D^B}{E_p \cos \theta + E_D}
\]
as the **geometric blue balance function** of the color primary. In Equation (5),

\[ E(\theta) \] is the total intensity of the light emitted from the test patch. \( E^B(\theta) \) is the blue component of the RGB-code of the light emitted from the test patch, as defined earlier. The last term in Equation 5 is gotten by substituting Eqs. 1 and 2 into the middle term. Equation 5 is the relative intensity of the blue primary in the light emitted from the test patch. We define a **geometric green balance function**, \( \Lambda^G(\theta) \) and a **geometric red balance function**, \( \Lambda^R(\theta) \), analogously, and refer to them collectively as geometric chromaticity balance functions. Note that when the light sources have the same RGB-chromaticities, the geometric chromaticity balance functions are all constant, independent of \( \theta \).

**Achromatic setting.** Suppose that the observer views a matte test patch in a scene illuminated by a combination of blue diffuse and yellow punctate sources. The angle between the normal to the test patch and the punctate light direction is \( \theta \). The observer is asked to adjust the chromaticity of the test patch without changing the total intensity until it looks achromatic. We denote his achromatic setting as a function of \( \theta \), by

\[ \hat{E}^B(\theta), \hat{E}^G(\theta), \hat{E}^R(\theta) \].

We define the **observer’s geometric blue balance function** by,

\[ \hat{\Lambda}^B(\theta) = \frac{\hat{E}^B(\theta)}{\hat{E}(\theta)}. \quad (6) \]

The observer’s geometric red and green balance functions are defined similarly. If the observer were perfectly color constant, then \( \left( \hat{E}^R(\theta), \hat{E}^G(\theta), \hat{E}^B(\theta) \right) \) would coincide
with \( (E^R(\theta), E^G(\theta), E^B(\theta)) \), \( \hat{\Lambda}^B(\theta) \) would be identical to \( \Lambda^B(\theta) \), \( \hat{\Lambda}^G(\theta) \) identical to \( \Lambda^G(\theta) \), and \( \hat{\Lambda}^R(\theta) \) identical to \( \Lambda^R(\theta) \).

By means of the color adjustment task just described, we can measure the observer’s geometric chromaticity balance functions and compare them to the theoretical ones for an achromatic Lambertian surface in the scenes we employ.
ANALYSIS AND RESULTS

Orientation Settings

For all observers, orientation settings deviated from the true values in both $\psi$ and $\varphi$, but these deviations were not large. Figure 8 shows one observer’s (BH) mean settings when the punctate source was positioned on the left. We have also plotted the best-fitting regression lines to BH’s settings. All other observers’ settings were similar to BH’s. We report the regression coefficients for all observers in Table 1.

We tested whether the orientation of the test patch in $\psi$ and $\varphi$ direction had a significant effect on observers’ orientation settings. As is evident in the plots, the main effect of orientation was significant for all observers for both directions ($p<0.0001$ in both $\psi$ and $\varphi$ directions). With the exception of subject MM in the $\psi$ direction, we found no significant interaction between perceived test patch orientation and punctate light source position (LEFT or RIGHT) for both directions ($\psi$ direction: $p=0.206, 0.637, 0.304$ for BH, MD and RG respectively, $p=0.01$ for MM; $\varphi$ direction: $p=0.852, 0.39, 0.07, 0.928$ for BH, MD, MM, an RG respectively). This implies that for all but one observer the position of the light source (LEFT or RIGHT) had no significant effect on how observers made their orientation settings.

Figure 8: Results: Orientation settings

We fit a linear model to the orientation settings ($\hat{\psi}_T$ vs. $\psi_T$ and $\hat{\varphi}_T$ vs. $\varphi_T$) separately for each observer. The estimated regression coefficient $a$ (intercept) is in units
of degrees, the regression coefficient $b$ (slope) is unitless. We report $p$ values for hypothesis tests against the corresponding veridical value (0 for $a$, 1 for $b$). In the $\phi$ direction slopes were significantly different from 1 for subject MD (punctate on RIGHT: $p<0.001$) and subject MM (RIGHT: $p<0.001$). All other subjects’ slopes in this direction were not significantly different from 1 for both punctate source positions (LEFT: $p=0.934$, 0.037, 0.148 for BH, MM, and RG respectively; RIGHT: $p=0.834$, 0.01, 0.125 for BH, MD and RG respectively). The intercepts in the $\phi$ direction were significantly different from 0 for all observers ($p<0.001$) except for observer RG (LEFT: $p=0.782$, RIGHT: $p=0.042$).

In the $\psi$ direction slopes were significantly different from 1 for all observers ($p<0.001$), except observer BH ($p=0.009$, RIGHT: $p=0.005$). The intercepts in the $\psi$ direction were significantly different from 0 for all observers ($p<0.002$) except MM (LEFT: $p=0.398$, RIGHT: $p=0.038$).

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**Table 1: Results: Orientation setting regression coefficients**

We will use the observers’ own estimates of orientation in analyzing the effect of orientation on achromatic settings. However, it would not alter our conclusions in any important respect if we used veridical orientations instead.

**Achromatic settings**

If an ‘ideal’ Lambertian observer could estimate the parameters that describe the orientation and chromaticity of the test patch, then his or her achromatic setting would
match the geometric chromaticity balance function given in Equation (5).

However, suppose that the observer is effectively using Equation 5 to arrive at estimates of surface color appearance but the observer’s estimates of some or all of the parameters in the equation were in error. Then the achromatic setting would not match the geometric chromaticity balance function. We let $\hat{\phi}, \hat{E}_p^B, \hat{E}_D^B$ etc. denote the observer’s estimates of $\phi, E_p^B, E_D^B$ etc. We assume these estimates, although unknown, do not change over the course of the experiment. As mentioned before, by recording the observer’s achromatic setting over different values of the angle of incidence, $\theta$, we can determine the observer’s estimates of the parameters in Equations (5) and (4).

**Geometric Blue balance function**

The chromaticities of the light sources in the rendered scenes differ only in the blue-yellow balance; therefore we are primarily interested in the blue-yellow component of the observers’ achromatic settings and we first consider the geometric blue balance function.

Equation 5 can be rewritten as,

$$\Lambda^B(\theta) = \frac{\pi^B \cos \theta + \delta^B \Delta}{\cos \theta + \Delta},$$

where the variables $\pi^B$, $\delta^B$ and $\Delta$ were defined above. An observer’s visual system can compute what the blue balance of a gray surface should be if estimates of the parameters in Equation (7) are available. However if the observer’s estimates of the parameters are in error, the achromatic settings would differ from the predicted ones. Let $\hat{\pi}^B$, $\hat{\delta}^B$, $\hat{\Delta}$ and $\hat{\theta}$ denote the observer’s estimates of the parameters in Equation (5), then
\[
\hat{\Lambda}^B(\theta) = \frac{E^B_{\text{ver}}(\hat{\theta})}{E(\hat{\theta})} = \frac{\hat{\mathbf{n}}^B \cos \hat{\theta} + \hat{\mathbf{d}}^B \hat{\Lambda}}{\cos \hat{\theta} + \hat{\Lambda}}.
\] (8)

The observer’s estimate of the angle of incidence \(\hat{\theta}\) depends on his or her estimate of the orientation of the test patch \((\hat{\psi}_r, \hat{\phi}_r)\) and his or her estimate of the direction to the punctate source \((\hat{\psi}_p, \hat{\phi}_p)\) through Equation (4). Note that the observers explicitly estimated the orientation of the test patch \((\hat{\psi}_r, \hat{\phi}_r)\) by performing the orientation task.

Suppose that we hold the lighting conditions constant, in particular the parameters
\[
\Theta = \{\psi_p, \varphi_p, \pi^B, \delta^B, \Delta\},
\] (9)
and vary the orientation of the surface by varying \(\psi_r\) and \(\varphi_r\) as we do in the experiment.

Figure 9a shows the geometric Blue balance function \(\Lambda^B\) plotted with respect to the angles \(\psi_r\) and \(\varphi_r\) assuming the veridical values of the lighting parameters, \(\Theta\). Now suppose that the estimates of the lighting parameters \(\hat{\Theta} = \{\hat{\psi}_p, \hat{\phi}_p, \hat{\pi}^B, \hat{\delta}^B, \hat{\Delta}\}\) are in error, what kind of distortions would those errors introduce? Misestimating the direction to the punctate source shifts both curves without much effect on their curvatures (Figure 9b).

When the test patch is oriented such that it faces the punctate source as directly as possible, that is \(\psi_r = \psi_p\) (after a \(\psi\) rotation) or \(\varphi_r = \varphi_p\) (after a \(\varphi\) rotation), it receives the maximum possible amount of light from the yellow punctate source. However the blue content of the mixture of light falling upon it remains fixed, hence the blue balance, \(\Lambda^B\), assumes its minimum.
More rigorously, the extrema of the function $\Lambda^b$ are found by taking its derivative with respect to $\psi_T$ and $\varphi_T$, and then equating it to zero, which yields

$$\begin{align*}
\psi_T^* &= \psi_p, \\
\varphi_T^* &= \arctan\left(\frac{\tan \varphi_p}{\cos \psi_p}\right) \approx \varphi_p \quad \text{(for small $\psi_p$)}.
\end{align*}$$

(For $\psi_p = \pm 15^\circ$, $\varphi_T^* = 30.87^\circ$, only slightly different from $\varphi_p = 30^\circ$.) Note that $\psi_T^*$ and $\varphi_T^*$ correspond to minima for $\pi^b < \delta^b$, and to maxima for $\pi^b > \delta^b$ (see Equation (13) below).

If the Lambertian observer misestimated the punctate source direction, his or her achromatic settings would reveal this because his or her geometric blue balance function $\hat{A}^b$ would shift and have its minimum at roughly the estimated direction to the punctate source ($\hat{\psi}_p, \hat{\varphi}_p$).

---

Figure 9: Blue balance: Right answer and possible errors.

Errors in estimating the parameters $\pi^b$, $\delta^b$, and $\Delta$ shift the curves up or down and increase or decrease their curvatures (Figure 10). When $\hat{\pi}^b = \hat{\delta}^b$, the geometric blue balance function becomes a constant. This is because if the blue balance of the punctate source ($\hat{\pi}^b$) and diffuse source ($\hat{\delta}^b$) were the same, rotating the test patch would not affect the overall chromaticity balance of the light reaching the gray test patch. Therefore the geometric blue balance function would become a constant. $\hat{\Lambda}^b$ is constant also when $\hat{\Delta} = 0$ or $\hat{\Delta} \to \infty$, that is if the observer estimates that the scene is illuminated either by only a punctate source or by only a diffuse source. However, since veridical values are such that $\pi^b \neq \delta^b$ and $\Delta$ is not 0 or infinity, if the observer’s geometric balance function
is a constant then the implication is that the observer does not discount the perceived orientation of the test patch for its color.

Figure 10: Possible errors

Figure 11 shows the empirical geometric Blue balance functions for all 4 observers. As mentioned above, if an observer were perfectly color constant, then his or her data would fall on the theoretical curve of the geometric Blue balance function $\Lambda^B$. On the other hand, if the observer were completely ignoring the orientation of the test patch in his or her achromatic judgment, than the ratio would be constant.

Figure 11: Results: Blue balance vs. perceived angle

It is clear that observers take the orientation of the test patch into account and have some degree of color constancy, although the constancy is not perfect. A comparison of the patterns of data to the family of $\Lambda^B$ curves in Figures 9 and 10 suggests that observers make settings that are indistinguishable from those of a Lambertian color constant observer who discounts the perceived orientation for estimating color, but who does so using incorrect estimates of the lighting parameters $\Theta$.

We explicitly measured observers estimates of $\psi_T$ and $\varphi_T$. We now use a maximum likelihood fitting procedure to estimate values of the lighting parameters $\Theta = \{\psi_p, \varphi_p, \pi^B, \delta^B, \Delta\}$ that best accounted for each observer’s data separately. These estimates are reported in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Achromatic settings: Maximum likelihood estimates of the lighting parameters</th>
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First note that observers’ punctate source direction estimates were close to the veridical values. We tested the hypothesis that the observer’s estimate of the punctate source direction \( \hat{\psi}_p, \hat{\phi}_p \) was equal to the veridical values by means of a nested hypothesis test (Mood, Graybill and Boes, 1974, pp. 440). We nested the hypothesis that \( \hat{\psi}_p = \pm 15 \) and \( \hat{\phi}_p = 30 \) (their true values) within a model in which they were free to vary. We fit both models to the data by the method of maximum likelihood with other parameters allowed to vary freely. The log likelihood of the constraint model (denoted by \( \lambda_0 \)) must be less than or equal to that of the unconstraint model (denoted by \( \lambda_1 \)). Under the null hypothesis, twice the difference in log likelihoods is asymptotically distributed as a \( \chi^2 \)-variable

\[
2(\lambda_0 - \lambda_1) = \chi^2. \tag{11}
\]

We use this result to test whether observer’s estimates \( \hat{\psi}_p, \hat{\phi}_p \) were significantly different from the true values. We summarize the values of the observers’ estimates \( \hat{\psi}_p, \hat{\phi}_p \) in Table 2 along with the corresponding \( p \) values. None of the observers’ punctate source direction estimates were significantly different from their true values under all conditions. All observers’ estimates of the punctate source direction were very close to the veridical (Table 2).

We next examined whether the observers were accurately estimating the Blue balance of the punctate and diffuse sources, \( \hat{\pi}^B \) and \( \hat{\delta}^B \), and the diffuse-punctate balance \( \hat{\Delta} \). We tested the hypothesis that the observer’s estimates were equal to the true values \( \pi^B = 0 \), \( \delta^B = 0.66 \), and \( \Delta = 0.37 \). We nested the hypothesis that the parameters were equal to the true values within a model in which they were free to vary. We separately
tested whether the observers did not discount the orientation at all, that is, whether the geometric Blue balance function was a constant.

We rejected the hypothesis that $\left(\hat{\pi}^B, \hat{\delta}^B, \hat{\Delta}\right) = (0, 0.66, 0.37)$ for all observers except observer BH for the punctate-on-the-right condition ($p=0.036$; all other $p$-values are reported in Table 2). We also rejected the hypothesis that the geometric balance function was a constant for all observers ($p<0.0001$ in all cases). These results imply that the observers indeed discount the orientation. However the discount is not exactly equal to the predictions of the model. The reason for the mismatch is that the observers’ estimates of the chromaticity balances of the punctate and diffuse sources, and of the diffuse-punctate balance are in error. All observers slightly overestimated the Blue balance of the punctate source, $\hat{\pi}^B > \pi^B$ (veridical value is $\pi^B = 0$), and misestimated the Blue balance of the diffuse source $\delta^B$, and the diffuse-punctate balance $\Delta$. The values are reported in Table 2.

**Red-Green balance**

We also examined the red-green balances of the achromatic settings. We define $\hat{\Lambda}^{R/G} = \frac{E_M^R}{E_M^G}$ as the red-green balance. We present only one observer’s (MM) settings in Figure 12. As expected, the red-green balance of achromatic settings did not change systematically with changes in the orientation of the test patch. All other observers’ results were similar.

Figure 12: Results: Red-Green balance of MM
A neutral light source control

As the test patch rotates away from the yellow punctate light source not only the relative blue content of the patch increases but also the overall intensity of the light from the test patch decreases (as described for Lambertian surfaces). What if these two events are confounded, that is, what if subjects simply assume that darker objects appear more blue? We verified that this is not the case by letting one observer (MD) run an extra session in the experiment where the punctate and diffuse light sources were neutral (\(E^P_R = E^P_G = E^P_B\) and \(E^D_R = E^D_G = E^D_B\), \(\delta^B = \pi^B\)). We found no effect of orientation on his achromatic settings, neither for blue-total balance (Figure 13) nor for red-green balance. This result is consistent with Equation (7), and indicates, for example, that observers do not simply increase the blue content of a gray surface as its total intensity decreases.

---

**Figure 13:** Blue-Total balance under Neutral lights.

Geometric discounting index

We can quantify the observers’ performance in discounting the perceived orientation for perceived color by, first of all, noting how close his or her estimates of light source direction are to the true direction. This measurement, though, does not reflect errors in the observer’s estimates of the remaining lighting parameters. We define a geometric discounting index that effectively compares the curvature of the observer’s geometric balance function at its minima,

\[
DI = 1 - \frac{|\kappa - \hat{\kappa}|}{\kappa + \hat{\kappa}},
\]

where
\( \kappa = \frac{K^\psi + K^\phi}{2}, \ \hat{\kappa} = \frac{\hat{K}^\psi + \hat{K}^\phi}{2}, \)

and

\[
\begin{align*}
K^\psi &= \frac{\partial^2 \Lambda}{\partial \psi_T^2} \bigg|_{\psi_T = \psi_T^*} \\
K^\phi &= \frac{\partial^2 \Lambda}{\partial \phi_T^2} \bigg|_{\phi_T = \phi_T^*} .
\end{align*}
\]  

(13)

are the curvatures at the minima. \( DI \), is a measure of how much the observer’s geometric blue balance function is curved compared to the theoretical one. A complete lack of curvature (\( \hat{\kappa} = 0 \)) corresponds to the case where the observer’s achromatic settings are unaffected by angle. Then \( DI = 0 \) and we would conclude that the observer’s color estimates are not affected by perceived surface orientation. If, in contrast, \( \kappa = \hat{\kappa} \), then \( DI = 1 \). Note that \( DI \) is a composite measure of all the parameters in the parameter space \( \hat{\Theta} = \{ \hat{\psi}_p, \hat{\phi}_p, \hat{\kappa}^\psi, \hat{\kappa}^\phi \} \). However, since the estimated parameters \( \hat{\psi}_p, \hat{\phi}_p \) were close to veridical, the errors are effectively due only to the misestimated chromaticity balances of the punctate source and diffuse source and the overall diffuse-punctate balance. The values of \( DI \) varied between 0.29 and 0.82 (\( DI = 0 \): no discount; \( DI = 1 \): perfect discount).

The discounting indices are reported in Table 3 and in the legends of Figure 12.

| Table 3: Discount Indices |
CONCLUSION

When a scene is illuminated by punctate and diffuse light sources differing in chromaticity, the chromaticity of the light absorbed by any patch or surface depends upon its orientation. We report one experiment in which observers were asked to view rendered three-dimensional scenes, binocularly. The lighting in these scenes was composite, a mixture of a yellow punctate light source and a blue diffuse. On each trial, observers adjusted the chromaticity of a test patch to be achromatic (achromatic setting task) and then adjusted a gradient probe to match the orientation of the patch (orientation task). We varied the orientation of the test patch randomly across trials. The location of the punctate source could be either LEFT or RIGHT, and was fixed for a given session.

We found that observers perceived the orientation of a Lambertian test patch placed in the scene nearly veridically, and that observers systematically take the perceived orientation of the test patch into account in making achromatic settings. However, their settings do not match the settings of an observer who has knowledge of the parameters of the composite lighting model (the location of the punctate light source and the chromaticities and intensities of the diffuse and punctate light sources).

We refit the observers’ data on the assumption that the observer correctly discounted the illumination arriving at the test patch in making achromatic settings, but that, in doing so, he or she made use of estimates of parameters of the composite lighting model that were in error. We found very good agreement between this model of the observer’s performance and our data. All of the observers’ estimates of the direction to the punctate source were close to veridical. The deviations from the model were mainly
due to the observers’ failure to use correct estimates of the chromaticities of the punctate source and of the diffuse source, and the ratio of intensity of the diffuse source to the intensity of the punctate source.

It has been shown that the perceived geometry of a scene influences the lightness (perceived albedo) of surfaces (Gilchrist 1977, 1980; Gilchrist et. al. 1999; Boyaci et. al. 2003). There are also studies which show that the perceived color of surfaces is influenced by the spatial arrangements permitting mutual illumination (Bloj et. al. 1999; Doerschner et. al. under review). Since we used several different orientations of the Lambertian test patch, we were able to parametrically determine how observers’ surface color estimates are influenced by surface orientation in scenes with composite light models. Observers do take into account the three-dimensional structure of scenes and the lighting model of the scene in arriving at estimates of surface color. Their shortcomings are predominantly consistent with failures to estimate the parameters of the lighting model correctly.

In conclusion, our results indicate that the observer, in effect, develops a model or estimate of the spatial distribution and chromaticities of light sources in a scene as part of color visual processing. Maloney & Yang (2003; Maloney, 1999, 2002) review previous work related to the hypothesis that the visual system develops an estimate of illuminant chromaticity and it is natural to extend this “illuminant estimation hypothesis” to include explicit estimation of the spatial distribution of light sources as well.

Our results also indicate that the observer’s estimates of light source chromaticity and spatial distribution of light sources can be markedly in error. The estimates of lighting parameters that we derived from each observer’s performance are very similar in
spirit to Brainard’s notion of an “equivalent illuminant” (Brainard, 1998) and they are a natural generalization of his idea to include spatial factors.

In pilot experiments, we examined scenes illuminated by a mixture of a yellow punctate source and a blue diffuse source and with few additional objects beyond the test patch and the ground plane. We found no effect of test patch orientation on its perceived surface color (though its orientation was perceived nearly veridically). Only when we inserted a faint, dark blue background and added more cubic objects to the scene, were we able to find the effect (none of the four observers who completed the experiment in its final form had also run the former versions). We conjecture that the added objects and background served as effective cues to the illuminant that allowed the visual system to estimate (if only imprecisely) the parameters describing the lighting model. It would be of interest to determine what structures in the three-dimensional scenes carry the information about the composite light model that the visual system makes use of.

We do not mean to suggest that observers are aware of their lighting models or of the cues that signal it (c.f. Rutherford & Brainard, 2002). Franz Kafka (1911) described the lighting in his room as “The lights and shadows thrown on the walls and the ceiling by the electric lights in the street and the bridge down below are distorted, partly spoiled, overlapping, and hard to follow. When they installed the electric arc-lamps down below and when they furnished this room, there was simply no housewifely consideration given to how my room would look from the sofa at this hour without any lights of its own.” Unlike his room, our scenes were designed carefully and with “housewifely consideration”. Yet, looking at the scenes of this experiment the observers did not have a better understanding of the purposes of the “creator”: when asked after the experiment,
observers were not even aware that the punctate source changed its position from session to session, or that there was a \textit{blue} background. Their visual system simply took care of the “overlapping, and hard to follow details” for them.
ACKNOWLEDGMENTS

This research was funded in part by Grant EY08266 from the National Institute of Health. HB and LTM were also supported by grant RG0109/1999-B from the Human Frontiers Science Program. We thank Michael Landy for comments on earlier drafts and David Brainard for comments on this work in poster form.
REFERENCES


Table 1: Results: Orientation setting regression coefficients. We fit a linear model to the orientation settings ($\hat{\psi}_T$ vs. $\psi_T$ and $\hat{\phi}_T$ vs. $\phi_T$) separately for each observer. The estimated regression coefficient $a$ (intercept) is in units of degrees, the regression coefficient $b$ (slope) is unitless. We report $p$ values for hypothesis tests against the corresponding veridical value (0 for $a$, 1 for $b$). We report exact $p$ values when the values are larger than 0.001. With a Bonferroni correction for 16 tests per observer, the significant level corresponding to an overall Type I Error rate of 0.05 for each subject is 0.0031. Values whose corresponding $p$ values fall below this cutoff are marked with an asterisk.
<table>
<thead>
<tr>
<th>punctate source position: LEFT</th>
<th>punctate source position: RIGHT</th>
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<tbody>
<tr>
<td>((\hat{\psi}_p, \hat{\phi}_p))</td>
<td>((\hat{\pi}^B, \hat{\delta}^B, \Delta))</td>
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<tr>
<td>((\hat{\psi}_p, \hat{\phi}_p))</td>
<td>((\hat{\pi}^B, \hat{\delta}^B, \Delta))</td>
</tr>
<tr>
<td>veridical ((-15, 30))</td>
<td>((0, 0.66, 0.37))</td>
</tr>
<tr>
<td>BH ((-23.1, 36.6))</td>
<td>((0, 0.33, 1.39)^*)</td>
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<tr>
<td>(p=0.011)</td>
<td>(p&lt;0.001)</td>
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<tr>
<td>MD ((-22.4, 28.8))</td>
<td>((0.1, 0.38, 1.87)^*)</td>
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<tr>
<td>(p=0.63)</td>
<td>(p&lt;0.001)</td>
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<tr>
<td>MM ((-15.8, 39.3))</td>
<td>((0.2, 1, 0.03)^*)</td>
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<tr>
<td>(p=0.734)</td>
<td>(p&lt;0.001)</td>
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<tr>
<td>RG ((-54.4, 39.5))</td>
<td>((0.2, 0.31, 2.15)^*)</td>
</tr>
<tr>
<td>(p=0.029)</td>
<td>(p&lt;0.001)</td>
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</table>

**Table 2: Achromatic setting: Maximum likelihood estimates of lighting parameters.** We report the maximum likelihood estimations of the punctate source direction \((\hat{\psi}_p, \hat{\phi}_p)\) and the lighting parameters \((\hat{\pi}^B, \hat{\delta}^B, \hat{\Delta})\). The parameter \(\hat{\pi}^B\) is the blue balance of the punctate source, \(\hat{\delta}^B\) is blue balance of the diffuse source and \(\hat{\Delta}\) is the ratio of the intensity of the diffuse source to the intensity of the punctate. For each observer, we tested the hypotheses that \((\hat{\psi}_p, \hat{\phi}_p)\) and \((\hat{\pi}^B, \hat{\delta}^B, \hat{\Delta})\) are equal to the veridical values and report exact \(p\) values for the tests when the values are larger than 0.001. With a Bonferroni correction for 40 tests (4 observers, 5 parameters, two punctate source positions), the significant level corresponding to an overall Type I Error rate of 0.05 is 0.00125. Values whose corresponding \(p\)-values fall below this cutoff are marked with an asterisk. All observers’ punctate source direction estimates were not significantly different than the veridical values. However in contrast with their success in estimating the punctate source direction, observers misestimated the other lighting parameters \((\hat{\pi}^B, \hat{\delta}^B, \hat{\Delta})\). The
deviations from the veridical values were significant (except for observer BH when the punctate source was on the right). Those deviations in the lighting parameter estimates result in failure to discount perceived orientation of the test patch for its perceived color exactly as the model predicts.
Table 3: Discounting Indices. We define a discounting index $DI$ to quantify how well observers discounted the effective illumination in the experimental scenes. $DI$ is a comparison of how the observers’ achromatic settings vary with perceived test patch orientation and how they should vary if the observer is correctly discounting the effective illumination on the test patch. The exact form of $DI$ is given in the text. A zero value of $DI$ means no discounting (the achromatic setting is unaffected by perceived orientation). A value of 1 corresponds to perfect discounting. All observers partially compensated for changes in effective illumination due to changes in test patch orientation.

<table>
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<th>punctate source position: LEFT</th>
<th>punctate source position: RIGHT</th>
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<td>0.27</td>
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<tr>
<td>RG</td>
<td>0.2</td>
<td>0.43</td>
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**FIGURE LEGENDS**

*Figure 1: An Achromatic Lambertian Surface.* The intensity of the light, that is absorbed by a surface is proportional to the cosine of the angle $\theta$ between the rays of light from the punctate light source and the surface normal, $\mathbf{n}$. A uniform diffuse light source contributes with a constant amount to the total intensity upon the surface. Light absorbed by a Lambertian surface is re-emitted uniformly in all directions; the intensity of light reaching the viewer does no depend upon viewing angle $\nu$, as long as the surface is visible. An achromatic Lambertian surface reflects all incoming light equally independent of its wavelength. For such an ideal surface, the ratio of the reflected light to the incoming light is determined by a wavelength-independent reflectance coefficient $\alpha$, the *albedo* of the surface.

*Figure 2: Sample Stimulus.* The stimuli were computer-rendered images of complex scenes. Each scene was rendered twice, from slightly different viewing points corresponding to the two eyes of the observers. The reader can fuse the left and center images (crossed fusion) or the center and right images (uncrossed-fusion). The virtual scene was illuminated by a yellow punctate light source positioned behind the observer either to his or her left or right and a blue diffuse light source. A *test patch* was located at the center of the scene. The test patch was the closest object to the observer in the scene (except the floor). By doing so, we eliminated any possible secondary illumination of the test patch by light emitted from other surfaces in the scene. Various additional objects, with a variety of surface reflectance properties (matte, shiny or transparent) were included in the
scene. The locations and properties of these objects were varied at random from trial to trial in the experiment.

Figure 3: Apparatus. Stimuli were displayed in a computer-controlled Wheatstone stereoscope. The left and right images of a stereo pair were displayed on the left and right monitors of the stereoscope. The observer viewed them by means of small mirrors placed in front of his or her eyes. Baffles placed to either side of the observers head blocked stray light from the monitors that might otherwise reach the eyes. In the fused image, the test surface appeared approximately 70 cm in front of the observer. This distance was also the optical distance to the screens of the two computer monitors, minimizing any mismatch between accommodation cues and other depth cues.

Figure 4: Spatial Coordinate System. We used a spherical coordinate system based on a Cartesian coordinate system to describe the geometry of the test patch and the punctate source. Both coordinate systems had their origins in the center of the test patch. Note that the center of the test patch was always in the same location throughout the experiment. In the Cartesian system \((x,y,z)\), the \(z\)-axis fell along the observer’s line of sight, the \(y\)-axis was vertical, the \(x\)-axis horizontal as shown. In the spherical coordinate system, a point in the three dimensional space is denoted by three numbers \((\psi, \varphi, r)\): \(r\) is the distance of the point from the origin. \(\psi\) is the angle between the observer’s line of sight (\(z\)-axis) and the projection of the point on the horizontal plane (\(xz\) plane), \(\varphi\) is the angle between the horizontal plane and the line connecting the origin and the point. The position of the punctate source is denoted by \((\psi_p, \varphi_p, r_p)\). The unit vector in the direction of the punctate source is denoted by \(\mathbf{p}\), the unit vector normal of the test patch is denoted by \(\mathbf{n}\). The
angle between the punctate direction and surface normal is calculated with the law of cosines: $\cos \theta = \mathbf{n} \cdot \mathbf{p}$.

**Figure 5: Monitor guns: Tests of Additivity.** We measured the red, green and blue gun luminances of our 21" Sony Trinitron Multiscan GDM-F500 monitors with a Pritchard PR-650 spectrometer. We first set each gun to half of its maximum possible intensity with the other two guns set to zero intensity. Luminance at half the maximum intensity is plotted separately across wavelength for each gun. The red, green and blue solid lines in Fig. 5A (left monitor) and Fig, 5B (the right monitor) correspond to the red, green ad blue guns. We then computed the sum of the three measured primaries (plotted as a gray solid line) and measured the luminance with all three guns simultaneously set to half their maximum intensities (black dashed lines).

**Figure 6: Orientations.** In each trial the test patch could appear with one out of ten orientations. Five of them were rotations of the fronto-parallel test patch in the $\psi$ direction, the other five were rotations in the $\phi$ direction. After a $\psi$ rotation, the orientation of the surface could be one of $\psi_T = \{-60^\circ, -45^\circ, -15^\circ, 15^\circ, 45^\circ\}$ when the punctate source was positioned on the left or $\psi_T = \{-45^\circ, -15^\circ, 15^\circ, 45^\circ, 60^\circ\}$ when the punctate source was positioned on the right, with $\phi_T = 0$. After a $\phi$ rotation, the orientation of the test patch could be one of $\phi_T = \{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ\}$

**Figure 7: Tasks.** On each trial, observers completed two tasks. A. The first one was the achromatic setting task: observers adjusted the chromaticity of the test patch until it appeared achromatic (“it looked as if it were cut out of an achromatic (gray) piece of paper.”) They adjusted the color of the test patch by pressing the arrow keys on a
computer keyboard. Pressing the “up” arrow key increased the blue content, while decreasing the yellow \((\text{red} + \text{green})\) content by the same amount. Pressing the “down” arrow had the opposite effect. Pressing the “right” arrow increased the red content and decreased the green content by the same amount, pressing the “left” arrow increased the green content and decreased the red content by the same amount. B. The second task was to estimate the orientation of the test patch. Observers adjusting a monocular gradient probe (presented to the right eye only). The probe consisted of two concentric circles and a stick placed at the center of the circles perpendicular to them. The observer’s task was to set the probe such that the stick was perpendicular to, and the circles were tangent to, the test patch.

Figure 8: Results: Orientation settings. This figure shows the orientation settings of observer BH when the punctate source was on the left. The graph on the left is for rotations of the test patch in the \(\psi\) direction, the graph on the right is for rotations in the \(\phi\) direction. The observer’s mean settings \(\hat{\psi}_T\) and \(\hat{\phi}_T\) for each angle \(\psi_T\) and \(\phi_T\) are represented by circles (blue for \(\psi_T\), red for \(\phi_T\)). The error bars represent 95% confidence intervals. The solid lines are the best linear fits, and the regression coefficients are given in the legends. All other observers’ results are similar to BH’s. Although there were deviations from veridical, those deviations were not large. The regression coefficients for all observers are given in Table 1.

Figure 9: Blue Balance Settings. A. The “right answer”. If the observer uses the correct values of the lighting parameters in the right hand side of Equation (7), then the geometric blue balance function, \(\Lambda^b\), calculated from his or her achromatic settings, would fall on the curves plotted in this figure. We plot \(\Lambda^b\) with respect to both \(\psi_T\) and \(\phi_T\) on the same
graphs. The blue solid line is the plot of $\Lambda^B$ with respect to $\psi_T$, the red one is with respect to $\phi_T$. The orientation of the test patch affects the geometric blue balance as follows: as the achromatic test patch rotates away from the direction of the yellow punctate it receives less and less yellow contribution (angle of incidence, $\theta$, increases, $\cos \theta$ decreases. See Equation 1). However, the blue contribution from the diffuse source does not change with this rotation. Therefore as the test patch rotates away from the punctate source, its blue balance increases. Conversely, as the test patch rotates closer to the direction of the punctate source its blue balance decreases and reaches a minimum when it faces the punctate source directly. In the experiment, however, the orientation of the test patch could vary either only in the $\psi$ direction or only in the $\phi$ direction. Hence $\Lambda^B$ has minima at $\psi_T = \psi_p$ ($\phi_T = 0$) and $\phi_T = \phi_p$ ($\psi_T = 0$).

**B. Errors in estimating punctate light direction.** What happens if the observers’ estimates of the parameters in Equation (7) are in error? Suppose that the observer’s estimate $\hat{\psi}_p$ of the direction to the punctate source is in error. If the observer based his or her settings on this or her erroneous estimate, then the minimum of the blue curve would be at $\hat{\psi}_p$ instead of the correct value, $\psi_p$, as shown in the upper plots. An error in the estimate of $\psi_p$ also affects the $\Lambda^B$ versus $\phi_T$ curve. The pattern of shifts when $\hat{\psi}_p < \psi_p$ and for $\hat{\phi}_p < \phi_p$ are shown in Figure 9B. The patterns when $\hat{\psi}_p > \psi_p$ and $\hat{\phi}_p > \phi_p$ are just the reverse.

**Figure 10: Errors in estimating other lighting parameters.** Under- or over-estimating the other three lighting parameters $\pi^B$, $\delta^B$, and $\Delta$ lead to systematic changes in the $\Lambda^B$ versus $\psi_T$ and the $\Lambda^B$ versus $\phi_T$ curves. They are discussed in the text.
**Figure 11: Results: Blue balance versus perceived orientation.** We plot all four observers’ geometric *blue* balance functions with respect to perceived orientation. The blue filled circles are the mean values of the *blue balance* of their achromatic settings at the mean perceived angle $\hat{\psi}_r$. The red ones are for $\hat{\phi}_r$. The solid lines are the best fitting curves according to the model described in the text. The small blue and red arrows point to the observers’ estimates of the punctate source direction. Notice that observers’ estimates of the direction to the punctate source are close to the correct values. The direction estimates were not significantly different from correct. Discounting indices are reported in legends. The flat black dashed lines correspond to observers’ settings if they did not compensate at all for orientation. Clearly observers are discounting the perceived orientation for perceived color, but the degree of discounting is not as large as the model predicts. Error bars are $\pm 2 \text{SE}$ of the mean (approximately a 95% confidence interval).

**Figure 12: Red-Green balance.** We checked observers’ achromatic settings for the ratio of the *red* content to *green* content. Since the *red-green* balance of the punctate and diffuse source is constant, independent of the orientation of the test surface, we did not anticipate any variation in the *red-green* balance of observers’ settings. This is what we found. We plot one observer’s (MM) results here. All other observers’ results were similar. Error bars are $\pm 2 \text{SE}$ of the mean (approximately a 95% confidence interval).

**Figure 13: Achromatic setting under neutral light.** As a control for the results, we repeated the experiment with *neutral* light sources ($E_p^R = E_p^G = E_p^B$ and $E_D^R = E_D^G = E_D^B$, $\delta^B = \pi^B$). Only one observer (MD) completed the control experiment. His results show no patterned change with changing orientation of the test patch. This lack of pattern indicates that he
does not simply add more blue content as the test patch rotates and gets darker. Error bars are $\pm 2 \, SE$ of the mean (approximately a 95% confidence interval).
\[ \psi_T = 3.3 + 1.06 \psi_T \text{ veridical} \]

\[ \phi_T = 4.8 + 0.98 \phi_T \text{ veridical} \]
Boyaci et al: Fig 10
Discount Index = 0.77

Discount Index = 0.8

Discount Index = 0.69

Discount Index = 0.82

Boyaci et al: Fig 11
Boyaci et al: Fig 11 (cont.)
$\hat{\Lambda}_{R/G}$ vs. $\hat{\psi}_T$ and $\hat{\phi}_T$
Boyaci et al. Fig. 13