COGNITIVE NEUROSCIENCE, 2015, http://dx.doi.org/10.1080/17588928.2014.1003181

Routledge
Taylor & Francis Group

# Revealing individual differences in strategy selection through visual motion extrapolation

**Jacqueline M. Fulvio[1], Laurence T. Maloney[2,3,4], and Paul R. Schrater[5,6]**

[1]Department of Psychology, University of Wisconsin - Madison, Madison, WI 53705 USA
[2]Department of Psychology, New York University, New York, NY 10003 USA
[3]Center for Neuroscience, New York University, New York, NY 10003 USA
[4]Institute for the Interdisciplinary Study of Decision Making, New York University, New York, NY 10003 USA
[5]Department of Psychology, University of Minnesota, Minneapolis, MN 55455, USA
[6]Center for Cognitive Sciences, University of Minnesota, Minneapolis, MN 55455, USA

Humans are constantly challenged to make use of internal models to fill in missing sensory information. We measured human performance in a simple motion extrapolation task where no feedback was provided in order to elucidate the models of object motion incorporated into observers' extrapolation strategies. There was no "right" model for extrapolation in this task. Observers consistently adopted one of two models, linear or quadratic, but different observers chose different models. We further demonstrate that differences in motion sensitivity impact the choice of internal models for many observers. These results demonstrate that internal models and individual differences in those models can be elicited by unconstrained, predictive-based psychophysical tasks.

***Keywords:*** Visual perception; Motion extrapolation; Internal models; Uncertainty; Individual differences.



Human vision is based on internal models of the external world. These representations allow us to overcome uncertainty, handle sparse or ambiguous information, and extract the information appropriate for many tasks. Internal models are particularly crucial for prediction. Trying to catch a hummingbird and a football require very different strategies. Considerable research has been devoted to understanding how internal models affect motor planning (Kawato, 1999; Miall & Wolpert, 1996; Wolpert, Ghahramani, & Jordan, 1995). However, the internal models that guide visual perception are less well understood.

In order to investigate the internal models used in perception, we developed a prediction-based task. Observers extrapolated the trajectory of a target through an occlusion region and predicted where it will reemerge (see Figure 1A). No feedback was given. No sensory information was available during occlusion, requiring observers to rely on a model to extrapolate the path of the target. Because no feedback was given, there was no single correct model they needed to use to extrapolate. Their extrapolation behavior allows us to characterize whatever internal model of motion they used.

We emphasize that we withheld feedback to avoid explicitly training observers to adopt a particular model during the experiment. There are infinitely many possible models that could describe the

Correspondence should be addressed to: Jacqueline M. Fulvio, Department of Psychology, University of Wisconsin – Madison, 461 Brogden Hall, 1202 West Johnson St, Madison, WI 53706. Email: fulvio@wisc.edu

The authors have no relevant financial or nonfinancial relationships to disclose.
This work was supported by the National Science Foundation under a Graduate Research Fellowship; The Office of Naval Research under Grant N00014-07-1-0937 (PRR); The National Institutes of Health under Grant NEI R01 EY015261 (PRR) and Grant NEI R01 EY01889 (LTM).

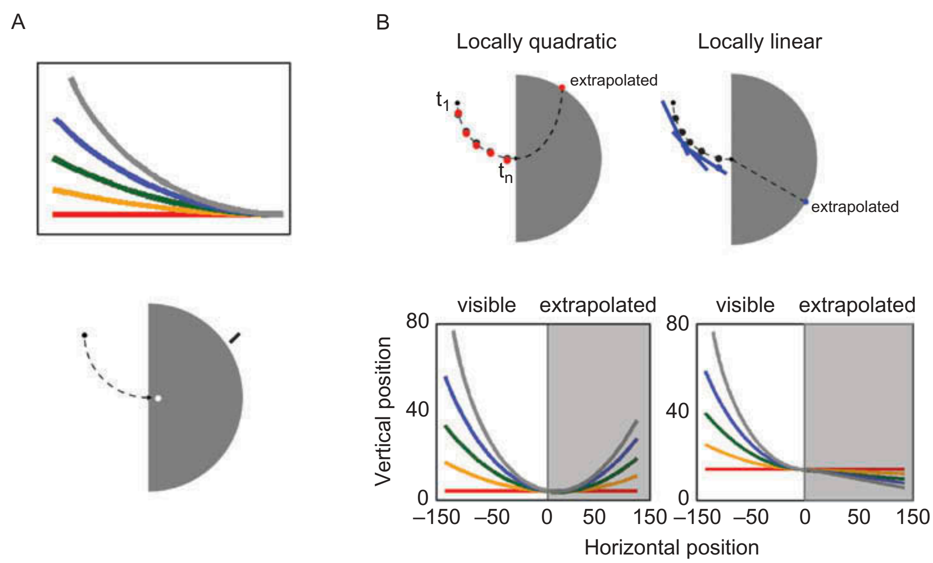


**Figure 1.** **A.** Schematic of the experimental task. Observers watched a target traverse one of five curved trajectories (top left) before disappearing behind a half-disk occluder. The trajectories were randomly-oriented on each trial. The observer had to predict where the target would reemerge from occlusion and indicate whether that location was above or below the tick mark that resided along the curved edge of the occluder (bottom left). No feedback was provided. **B.** Illustration of locally quadratic and locally linear extrapolation of the curved stimulus trajectories. We assume a Kalman filter model of extrapolation (see Methods and Results for details) in which observations of the motion are taken at each time step ($t_i$) and used to make a prediction for the subsequent time step ($t_{i+1}$) according to an assumed model (i.e., locally quadratic or linear). The two strategies yield very different patterns of performance: Locally quadratic extrapolation yields predicted endpoints with the same order as their respective curve inputs because of the curvature propagation, whereas locally linear extrapolation yields endpoints with the opposite ordering due to the propagation of the linear tangent to the visible curved trajectories.



extended motion of an object, but they can be parameterized by complexity (Spiegelhalter, Best, Carlin, & van der Linde, 2002) according to polynomial degree. Research on contour completion in human vision (Singh & Fulvio, 2005, 2007), computer vision (Horn, 1983; Kimia, Frankel, & Popescu, 2003; Mumford, 1994), and natural image statistics (Geisler & Perry, 2009) suggests low complexity models are used to fill in missing portions of sensory information. Two plausible motion models for our task are locally linear and locally quadratic (see Figure 1B). A key question is whether we find evidence for the use of these low complexity motion extrapolation models in our task or possibly other models. Such results would provide insight into the visual processes that contribute to how humans select and use models in extrapolating movement.

Another key issue is whether there are individual differences in internal model usage. We do not provide feedback or enforce accuracy, so observers may adopt the strategy of their choosing. Such behavior is not unprecedented—individuals adopt diverse strategies in other domains, such as when solving reasoning problems (Roberts, Gilmore, & Wood, 1997). Furthermore, in the context of static curved contour extrapolation, different observers vary the extent to which they incorporate curvature in their extrapolation yielding a range of behavior between locally linear and locally quadratic extremes, despite viewing the same stimuli (Singh & Fulvio, 2005). Finally, individual differences may reflect differences in visual processing, prior motion experience, or both. If different observers make use of different models, then it is important to determine what aspects of the task, and the observer, drive model selection.

# METHODS

## Psychophysical experiments

*Observers*

Eleven observers with normal or corrected-to-normal vision participated in two tasks: Motion extrapolation and discrimination. None was aware of the purpose of the study and all provided informed consent approved by the New York University human subjects committee prior to participation. Seven observers returned for a modified version of the motion extrapolation task.

*Apparatus*

The experimental apparatus included a 24” Sony GDM-FW 900 monitor with a display area of 48.2 cm x 30.8 cm, set to a resolution of 800 x 600 pixels at a vertical refresh rate of 160 Hz. The viewing distance was 36 cm.

## Extrapolation

*Stimuli*

Observers viewed a dot (“target”) travelling continuously along a circular arc of one of five curvatures before disappearing behind the midpoint of the straight edge of a half-disk occluder (see Figure 1A). The stimuli were chosen to minimize eye movements. Five values of curvature (defined as the inverse of the radius of a circular arc in pixels with larger values corresponding to greater curvature) were used: 0 (straight line; see also Pavel, Cunningham, & Stone, 1992), 0.0011, 0.0024, 0.0037, and 0.005. The occluder’s radius was 150 pixels (6.55 degrees of visual angle). The target’s radius was 10 pixels (0.437 degrees of visual angle) and traveled at a constant speed of 5 deg/s, so that the target’s trajectory spanned 121, 120, 116, 110, and 101 frames, respectively, yielding presentation times ranging from ~0.75 sec to ~0.63 sec. The trajectories were presented concave up, and randomly oriented between −5 and 5 degrees relative to the horizontal. Observers were encouraged to take short breaks every 100 trials to aid fixation and reduce fatigue.

Seven observers performed the same task with three additional levels of motion sub-sampling: 60% (2 of every 5 frames omitted), 50% (3/6 frames omitted), and 43% (4/7 frames omitted). During the blank intervals, $\Delta x$ was 2 pixels (60%), 3 pixels (50%), and 4 pixels (43%). Similarly, $\Delta t$ was 0.0125 sec (60%), 0.019 sec (50%), and 0.025 sec (43%). All other task details were identical to the continuous (100%) version.

*Task*

Observers fixated a small dot near the point of occlusion while the target traveled from the left side of the screen toward an occluder. After occlusion, a tone sounded and observers indicated by key press whether the target would re-emerge from occlusion above or below a tick mark 8 pixels (0.364 degrees of visual angle) in length displayed orthogonal to the opposite curved edge of the half-disk. Observers had to wait until the target reached the visible portion of its trajectory—early responses were not accepted. There was no other time constraint on response. No feedback of any kind was provided.

*Procedure*

The tick mark’s position was controlled by a staircase procedure (described in the Online Supplement). Five psychometric functions (one for each curve) were fit for each observer in order to determine the threshold extrapolation angle in degrees relative to the horizontal. Each was based on 200 trials.

*Software and apparatus*

The experiment utilized the Psychophysics Toolbox for MATLAB (Brainard, 1997). Psychometric functions were fitted as described in the Online Supplement.

## Discrimination

*Stimulus*

The stimulus consisted of a target that followed a trajectory with fixed start and end paths, but varied according to the transition (“kink”) between them (see Figure S1 and the Online Supplement for details of the stimuli and staircase procedure used). The smoothness of the “kink” was indexed by a parameter $\sigma_t$ (see Online Supplement). We measured observers’ ability to discriminate smoothness.

*Task*

Each trial contained two intervals, standard and test. The smoothness of the stimulus in the standard interval was fixed at $\sigma_s$. The smoothness of the stimulus in the test intervals varied from trial to trial under staircase control. Observers fixated a small dot in the center of the screen and observed both intervals. They indicated which interval had smoother motion by key press. Feedback was not provided.

One psychometric function (based on 300 trials) was fit per observer to determine the $\sigma_t$ that corresponded to the smallest difference between the test and standard stimuli with smoothness $\sigma_s$ that could be reliably discriminated. The variance in the threshold estimate provided a measure of the observer's (in)sensitivity to motion curvature.

## SIMULATED EXTRAPOLATION

The Kalman filter is an optimal recursive algorithm for state estimation (Kalman, 1960; Simon, 2006) in linear systems with Markov dynamics and Gaussian noise statistics. It combines a dynamics model with noisy measurements to produce estimates that maximize predictive accuracy.

The Kalman filter estimates a state vector $\hat{\mathbf{s}}_\mathbf{t}$ that includes the x and y positions of the object, its x- and y-velocities ($v_x$, $v_y$), and its x- and y-accelerations ($a_x$, $a_y$) at each time $t = 0,1,2\ldots$. The estimated state vector evolves under a dynamics model defined by a general linear process.

Extensive details of the modeling are outlined in the Online Supplement. Briefly, at each time step, a trajectory prediction is made

$$\hat{\mathbf{s}}_\mathbf{t} = \mathbf{A}\mathbf{s}_{\mathbf{t-1}} + \mathbf{w}_{\mathbf{t-1}} \quad (1)$$

by updating the linear extrapolation (prediction) from the internal representation of the target $\mathbf{s}_{t-1}$ at time *t-1* to the predicted state at the next moment in time, $\mathbf{s}_t$. This formulation allows us to use the same process in simulating multiple extrapolation strategies simply by varying the transition matrix, $A$, as described below.

The term $\mathbf{w}_{t-1}$ in Equation (1) represents the process uncertainty (noise) with $p(\mathbf{w}_t) \sim N(0, \mathbf{Q})$ with $\mathbf{Q}$ being a covariance matrix that adds uncertainty to either acceleration (locally quadratic) or velocity (locally linear), respectively:

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_v^2 \end{bmatrix} \quad (2)$$

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_v^2 \end{bmatrix} \quad (3)$$

The process noise is the observer's estimate of model error. Process noise can be interpreted as model error from a system identification viewpoint because an agent learning the system noise $\mathbf{Q}$ will estimate it from the prediction error.

The transition matrix $\mathbf{A}$ defines the specific extrapolation strategy by encoding assumptions about the target's dynamics process. Changing the parameters of $\mathbf{A}$ modify the complexity of the extrapolation model.

Locally quadratic extrapolation is defined by the $\mathbf{A}$ matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

which involves estimates of position, velocity, and acceleration in x and y, respectively. This model captures the curvature in a trajectory of a target undergoing constant acceleration in a gravitational field and has been previously implicated in human static contour extrapolation (Singh & Fulvio, 2005, 2007). Linear extrapolation is simpler but less flexible, with acceleration omitted. It involves state estimates of position and velocity processing in x and y, respectively:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Linear extrapolation behavior has been previously documented in motion prediction, even when the trajectory is visibly curved (Mrotek & Soechting, 2007). Schematics of these models are depicted in Figure 1B.

The prediction at each time step is, in effect, scored by estimates derived from trajectory measurements. At each time step, a target observation $z_t$ occurs via the following equation:

$$\mathbf{z}_\mathbf{t} = \mathbf{H}\mathbf{s}_\mathbf{t} + \mathbf{v}_\mathbf{t} \quad (6)$$

which represents the linear relationship between the target's true state and the observable parameters of that state, corrupted by measurement uncertainty. The matrix $H$ defines which parameters are perceived in the visible trajectory. We assume that observers' measurements are dominated by the velocity of the target's path for reasons described below (see Results). The $\mathbf{H}$ matrices for the two extrapolation strategies, locally quadratic and locally linear, respectively, are as follows:

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (7)$$

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Although the dynamics assumptions vary, both models take the same measurements from the visual data. The models only differ in that the locally quadratic model estimates acceleration "internally" and uses this estimate in extrapolation. The term $\mathbf{v}_t$ represents measurement uncertainty with $p(\mathbf{v}_t) \sim N(0, \mathbf{R})$ with $\mathbf{R}$, a covariance matrix (see Online Supplement for more detail).

At each time step $\mathbf{z}_t$ is combined with $\hat{\mathbf{z}}_t$ to update the representation and compute the next prediction via a series of update equations (see Online Supplement). We model occlusion and sub-sampling as the *absence* of measurements at those time steps—the position, velocity, and acceleration components become latent variables that are estimated without correction from direct observations. The prediction process continues via Equation (1) using the estimates from the last measurement made before occlusion.

# EVALUATING MODEL PREDICTIONS FOR HUMAN JUDGMENT DATA

To determine which model is the better predictor of each observer's extrapolation, we evaluated the *model evidence*—the marginal probability of an observer's thresholds under both models (Zucchini, 2000). We simulated performance under the two strategies for a broad range of measurement (**v**) and process (**w**) uncertainty combinations. This performance was used to compute the error $err_m(v,w) = \hat{\theta}_t - \theta_{t,m}(v,w)$ between the predicted endpoint angles $\theta_{t,m}$ for each of the five trajectories and the observer's extrapolated endpoint angles $\hat{\theta}_t$ for each uncertainty combination $(v,w)$ and each strategy $m$, thus making no assumptions about each observer's uncertainties. The model evidence was defined as the probability of the errors for model $m$ integrated over all noise parameter combinations (i.e., $p(\text{data}|\mathbf{v},\mathbf{w})$) assuming a flat prior, $p(\mathbf{v},\mathbf{w})$:

$$e_m = -\log \int_v \int_w p(err_m|\mathbf{v},\mathbf{w}) p(\mathbf{v},\mathbf{w}) dv\, dw \quad (9)$$

We approximated this integral numerically. Finally, the relative model evidence was computed as the marginal log-likelihood ratio, found by subtracting the evidence for the locally linear strategy from that of the locally quadratic strategy. Positive values indicate that the locally quadratic strategy better accounts for the observer's data.

# RESULTS

## Simulated extrapolation

We used the Kalman filter to simulate extrapolation performance via locally linear and locally quadratic models to provide a basis for comparison with human data. We assumed extrapolation is carried out as a statistical estimation problem in which predictions about the target's motion are made incrementally via local internal models as visual information becomes available (e.g., Kording, Tenenbaum, & Shadmehr, 2007; Wolpert et al., 1995).

We assumed observers obtain noisy measurements of the target's *velocity*. Evidence supports direction-selective motion detectors for localization of moving targets (Welch & McKee, 1985) during periods of occlusion (Grzywacz, Watamaniuk, & McKee, 1995; Watamaniuk, 2005; Watamaniuk & McKee, 2002). Once in motion, sensitivity to a target's position declines (Chung, Levi, & Bedell, 1996; Morgan, Watt, & McKee, 1983) and is largely impervious to positional noise, including random jitter (Badcock & Wong, 1990; Banton & Levi, 1993). Many studies have found insensitivity to stimulus acceleration when acceleration represents a change in speed, not direction (Gottsdanker, 1956; Schmerler, 1976; Watamaniuk & Duchon, 1992). We are not aware of any previous studies that have looked at sensitivity to acceleration corresponding to changes in direction with no concomitant change in speed.

We further assumed that observers choose the model most consistent with the noisy data collected while observing the target's motion. Given a polynomial model and noisy data, the Kalman filter recursively estimates the state of the moving target (i.e., the trajectory parameters) using an internal model (Equation (1)).

A key advantage of our formulation is that any differences in simulated predictions between the two strategies can be interpreted as due *strictly to a difference in the specific internal model* (the assumptions encoded by the **A** matrix). The patterns of performance resulting from the two models are distinct (see Figure 1B). Locally quadratic extrapolation yields predicted endpoints with the same order as their respective visible inputs due to propagation of velocity and acceleration. Locally linear extrapolation yields endpoints with the opposite ordering due to propagation of velocity only.

## Human extrapolation

We hypothesized individual differences would emerge in the choices of internal models used to achieve open-ended visual tasks such as ours, and that those models would be low in complexity. Figure 2A shows observers' threshold extrapolation endpoint angles in degrees relative to the horizontal (0 deg) with bootstrapped 95% confidence intervals for each observer. Individual differences are apparent—the extrapolated endpoints span a continuum of performance from a large increase in angular deviation from straight (0 deg) with visible trajectory curvature to much smaller decreases in angular deviation from straight. We note that although observers did not typically extrapolate accurately, the endpoint patterns clearly cluster into two groups across observers that are consistent with the orderings predicted by the locally quadratic and locally linear models as shown in Figure 1B. We used this result to qualitatively characterize observers based on their extrapolated endpoint ordering. Those who had at least three of five endpoints in the appropriate order were labeled quadratic extrapolators (1–6); the remaining (7–11) were labeled linear extrapolators.

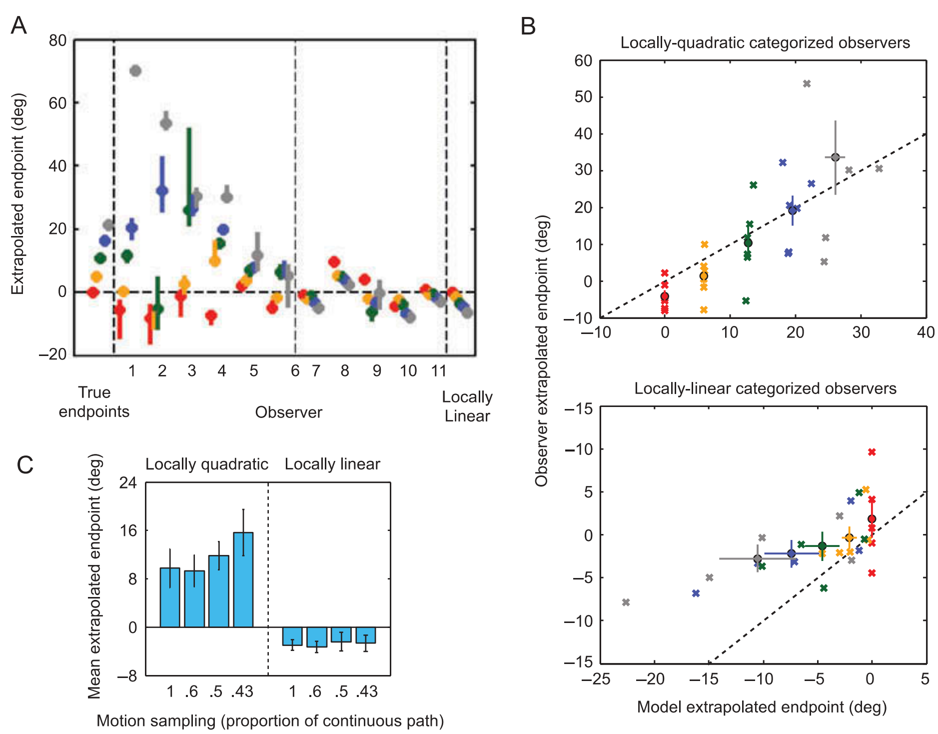


**Figure 2. A. Results for the extrapolation task.** Observer threshold extrapolation endpoint angles in degrees relative to the horizontal (0 deg) for each of the five motion trajectories. Error bars represent bootstrapped 95% confidence intervals on the threshold estimates. The true trajectory endpoints (ideal performance) are (far left) and the endpoints predicted by the locally linear model (far right) are depicted for comparison with the human data. Inspection of the observers' endpoints reveals a wide range of performance. Nevertheless, the data consists of two categories of observer performance consistent with the qualitative performance of the simulated locally quadratic and locally linear strategies depicted in Figure 1B. **B. Comparison between observer extrapolation and the extrapolation of the best-fitting model to the observer data.** Individual observers are plotted with "x" symbols and group means are plotted in circular symbols. The color scheme is the same as that used in **A.** There is strong agreement between the model predictions and the observer extrapolations, indicating that the models provide an accurate depiction of the human performance. **C. Summary of extrapolation performance on the sub-sampled dot motion conditions.** Four observers who used a locally quadratic model (2, 3, 5, 6) and three who used a locally linear model (7, 9,10) in the initial extrapolation task with fully-sampled motion were tested on three additional interleaved sub-sampled conditions. Each observer's threshold extrapolation endpoints were estimated and then averaged. Finally, the between-subject mean extrapolated endpoint was computed for each of the two groups to provide a summary measure of extrapolation behavior depicted in the graph. Performance across the two groups remains distinct, while performance within each group does not change across sampling conditions. Error bars correspond to the standard error of the mean.

Figure 2B shows the observers' extrapolated endpoints for each of the five trajectories using the same color scheme as in Figure 2A plotted against the extrapolated endpoints from their corresponding best-fitting model (i.e., locally quadratic (upper plot) or locally linear (lower plot) with fitted measurement and process uncertainty). There is agreement between the extrapolated endpoints indicating that the models are providing an accurate depiction of the human performance. The relationship between observer and model extrapolation is significant for both groups ($r_{locally\text{-}quadratic} = 0.71$, $p < .001$; $r_{locally\text{-}linear} = 0.64$, $p < .001$). We note that there is some bias in the locally-linear model predictions relative to the locally linear observers' data for the high curvature conditions. We hypothesize that this is due in part to a response bias in which observers are less willing to extrapolate well below the horizontal as would be predicted by a veridical locally linear extrapolation. Additionally, one observer was categorized as locally linear due to the reverse ordering of the extrapolated endpoints, even though the endpoints remained above the horizontal. As a result, the locally linear model has a very difficult time fitting these points, owing to the fact that it predicts negative values. Nevertheless, at the group level, both models appear to capture the respective observer behavior well.

Adopting the locally quadratic strategy should result in less biased but more variable extrapolation behavior owing to greater model complexity and uncertainty associated with the additional parameters. Indeed, the extrapolation performance of the locally quadratic categorized observers is more accurate ($t(8) = 2.3575$, $p < .05$). With respect to variance, the 95% confidence intervals indicate larger variability in the extrapolation of the locally quadratic characterized observers (4.77 deg) than in the extrapolation of the locally linear characterized observers (1.43 deg). This difference is significant ($F(5, 4) = 11.1267$, $p = .003$).

We next determined whether observers' choices of extrapolation strategies were affected by changes in local stimulus information. A subset of the observers (locally quadratic categorized observers 2, 3, 5, and 6 from Figure 2A and locally linear categorized observers 7, 9, and 10) participated in the extrapolation task a second time with three levels of sub-sampled dot motion. Sub-sampling reduces the amount of information available without changing the motion dynamics or artificially altering observers' inherent sensitivity to motion information. The bar plot in Figure 2C depicts the average threshold extrapolation angle across observers and stimulus path conditions for all four sampling conditions. Both groups (based on the initial strategy classification) show little change in extrapolation performance with the reduction in motion information. A repeated-measures ANOVA reveals there is a significant difference between the performance of the two classified groups ($F(1, 15) = 26.4$, $p = .0037$), but there is no effect of motion sampling on extrapolation behavior ($F(3, 15) = 0.68$, $p = .58$), nor is there an interaction between extrapolation strategy and motion sampling ($F(3, 15) = 0.67$, $p = .58$). The lack of effect of motion sub-sampling on performance is found even when only the locally quadratic observer performance is considered ($F(3, 9) = 1.43$, $p = .30$). We conclude that the human visual system consistently favors low complexity internal models, but individuals differ in the *specific* models adopted, even when they participate in the same task and are provided with identical instructions.

## Internal model selection

We next investigated the factors that might influence strategy choices. We considered two additional measures for each observer: (1) model evidence and (2) motion sensitivity.

We first quantified the classifications based on observers' endpoint distributions by the extent to which the model correctly predicts extrapolation behavior. The plot in Figure 3A shows the relative evidence (see Methods) of the two strategies in each of the 11 observers' extrapolation performance, plotted in the same order as Figure 2A. The results reveal good correspondence between the qualitative endpoint classification and the quantitative model evidence classification. The evidence for the first six observers is in favor of the locally quadratic strategy whereas the evidence for the final five observers is in favor of the locally linear strategy. Indeed, the relative evidence for three observers (1, 6, and 8) does not strongly favor one or the other strategy; however, the slight edge in all three cases is in the direction of the qualitatively assigned strategy. Therefore, according to this measure, the observer classifications that emerge are identical to the endpoint classifications.

We considered the relationship between individual motion sensitivity and strategy selection. One's sensitivity to motion information impacts the accuracy and reliability of visual estimation. We assume that observers estimate their own ability to estimate motion information and use that estimate in deciding which model to use. We further assumed that observers' thresholds measured in the discrimination task are good indices of observers' subjective

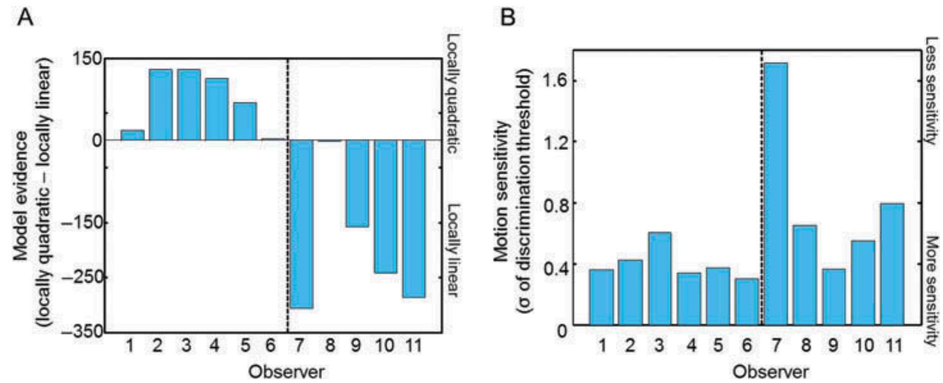


**Figure 3. A. Results of the model evidence analysis.** The relative evidence of the locally linear and locally quadratic strategies in each of the 11 observers' extrapolation performance, plotted in the same order as Figure 2A. Positive values favor locally quadratic extrapolation whereas negative values favor locally linear extrapolation. **B. Results of the motion sensitivity analysis.** The variability (σ) in the estimated discrimination threshold for accelerating motion is plotted for each of the 11 observers in the same order as **A.** Lower values mean greater sensitivity.

estimates. Thus, we hypothesized that the more variable an observer's discrimination threshold is, the poorer the observer's motion sensitivity, and the more likely the observer would be to adopt a simpler model. The plot in Figure 3B shows the motion sensitivity for each of the 11 observers plotted in the same format as Figure 3A. There is some tendency for locally quadratic classified observers (1–6) to have greater motion sensitivity (i.e., lower discrimination threshold variability) than locally linear classified observers (7–11); however, this relationship is far from perfect.

It is clear that some observers' strategies may be classified differently according to which measure is under consideration. This suggests that there are still additional factors not accounted for here that must be considered when predicting extrapolation strategies at the individual observer level (see Discussion below). At the group level, there is consistency among the three measures indicating that strategy choices are not idiosyncratic. For each measure (i.e., extrapolation endpoints, model evidence, and motion sensitivity), we classified observers into two groups (locally linear and locally quadratic) based on their performance. The extrapolation endpoint and model evidence classifications agree. When we categorize observers according to their motion sensitivity (using the discrimination threshold variability measure; Figure 3B) and then consider the mean extrapolation error for the two groups (left plot, Figure 4), we find that observers categorized as locally linear (i.e.,

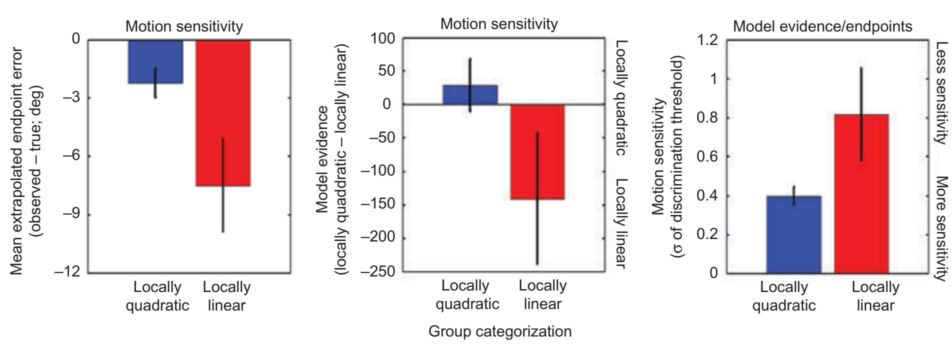


**Figure 4. Cross-measure comparison.** Left plot: Average extrapolated endpoint errors for the locally quadratic and locally linear classified groups according to motion sensitivity measure (locally quadratic with lower discrimination threshold variability: Observers 1–2, 4–6, 9; locally linear with higher discrimination threshold variability: Observers 3, 7, 8, 10, 11 from Figure 2A). Locally quadratic observers classified according to the sensitivity measure are generally more accurate in their extrapolation performance as expected. Middle plot: Average relative model evidence for the locally quadratic and locally linear classified groups according to motion sensitivity measure (same observer groups as in the left plot). The average relative model evidence for the two classified groups is in the expected direction, tending to favor the model consistent with the classification on average. Right plot: Average motion sensitivity defined as the mean variability in discrimination threshold for the locally quadratic and locally linear classified groups according to model evidence/extrapolated endpoint measures (locally quadratic: 1–6; locally linear: 7–11). Locally linear classified observers according to these measures have approximately double the variability in discrimination threshold as the locally quadratic classified observers. Error bars correspond to ± 1 SEM.

observers 3, 7, 8, 10, 11 from Figure 2A) have an average error that is approximately twice the average error of observers categorized as locally quadratic (i.e., 1–2, 4–6, 9). Similarly, locally linear observers categorized according to discriminability threshold variability also clearly favor the locally linear model (middle plot, Figure 4). The final comparison we consider indicates that those observers categorized as locally linear according to the extrapolation endpoint/model evidence measures, have discrimination threshold variability that is approximately twice that of those categorized as locally quadratic according to these measures (right plot, Figure 4).

## DISCUSSION

The motion extrapolation task we developed takes advantage of the visual system's need to invoke internal models when sensory information is missing or occluded and when judgments about future events are required. By withholding feedback and not enforcing a specific strategy, we were able to elicit the visual system's internal models and uncover a range of performance across individuals, for the same task.

We demonstrate that for the same task, observers' strategies were well characterized by one of two simple models that could be used to extrapolate the trajectories. Qualitative classification based on observers' extrapolated endpoint distributions, and quantitative classification on the basis of model evidence, leaves approximately half of our sample classified as locally linear extrapolators and the remainder as locally quadratic extrapolators. These strategies allowed us to predict local features of the target motion.

We also tested whether changing the amount of available information by sub-sampling the target motion, without changing the task or goal, would impact performance. Switching strategies frequently is effortful and comes with a computational cost (Rogers & Monsell, 1995). Observer performance in our task was unaffected by changes to the local information suggesting that strategy choices are not driven by individual trials, but rather the broader task. It is an interesting, yet separate, question for future work to explore more deeply the relationship between quantity and quality of sensory information and how these characteristics interact with internal beliefs and models in visual estimation. A second interesting direction for future research is to compare performance in the dynamic task we used here to analogous tasks where observers are asked to estimate curvature in static contours and extrapolate them (e.g., Singh & Fulvio, 2005, 2007; Watt & Andrews, 1982; Wilson, 1985). Performance in the dynamic task may also be further explored to better understand the information used and limitations of processing of moving targets (Pavel et al., 1992; Welch & McKee, 1985).

The next step in this line of work requires investigating the factors that determine individual observers' strategy choices. We began this investigation by relating observers' motion sensitivity to their performance. Motion sensitivity impacts the accuracy of observers' trajectory estimates—reduced sensitivity increases sensory uncertainty and estimation error. These propagate with extrapolation in the absence of visible information to correct any errors. The performance of different complexity models is impacted in different ways: Simple models generate larger biases when they are not flexible enough to fit data, such as fitting a line to a curve. Complex models reduce bias, but increase variance because they can be too flexible, fitting noise in the observations. The optimal choice is one that minimizes this *bias-variance trade-off* (Bishop, 2006).

Our analysis suggests that motion sensitivity plays some role in strategy selection for many observers. The more complex model tends to be adopted when sensitivity is greater. Future research is needed to shed light on other factors involved in strategy selection. One possibility is that evaluation of one's own sensitivity differs from our experimental measure. Another possibility is that observers do not have appropriate confidence in their ability to estimate motion information, and in their extrapolation accuracy using different models. Confidence in one's own ability develops with experience through feedback, which we have purposely withheld here.

In sum, the goal of the work presented here is, first, to investigate internal models used by human observers in motion extrapolation, and second, to understand individual differences in those models and what may be related to those differences. Importantly, our results demonstrate consistent individual differences in a simple perceptual task, and moreover, that these differences have a *categorical structure.* Although it is simple and considered perceptual, motion extrapolation is a key task showing profound deficits in schizophrenia, for example (Kraus, Keefe, & Krishnan, 2009). These linkages suggest that motion extrapolation is an exemplar of a task involving NMDA-driven top-down feedback. The fact that there are reliable individual differences in this task extends to the broader community in cognitive neuroscience.

Second, we provide evidence that observers' extrapolation strategies are influenced by factors that plausibly should affect their choice. This provides a deeper basis for understanding where we would expect individual differences in strategy—in complex structured domains where flexible models would support better predictions given accurate data, but where simple models provide reliable (yet biased) predictions given less accurate data.

## Supplementary material

Supplementary (Figure 1/content) is available via the 'Supplementary' tab on the article's online page (http://dx.doi.org/10.1080/17588928.2014.1003181).

Original manuscript received 27 April 2014
Revised manuscript received 21 December 2014
First published online 30 January 2015



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