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Comparison of the Distortion of Probability Information in Decision Under Risk and an Equivalent Visual Task

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Abstract

Decision makers typically overweight small probabilities and underweight large probabilities. However, there are recent reports that when probability is presented in the form of relative frequencies, this typical pattern reverses. We tested this hypothesis by comparing decision making in two tasks: In one task, probability was stated numerically, and in the other task, it was conveyed through a visual representation. In the visual task, participants chose whether a "stochastic bullet" should be fired at either a large target for a small reward or a small target for a large reward. Participants' knowledge of probability in the visual task was the result of extensive practice firing bullets at targets. In the classical numerical task, participants chose between pairs of lotteries with probabilities and rewards matched to the probabilities and rewards in the visual task. We found that participants' probability-weighting functions were significantly different in the two tasks, but the pattern for the visual task was the typical, not the reversed, pattern.

Keywords

decision making under risk, probability, probability weight, experience, visual perception, prediction

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All decisions share the same formal structure. There are many possible outcomes (O_1, O_2, . . . O_n), only one of which will occur. There are possible actions that affect the probabilities (p_1, . . . p_n) of obtaining corresponding outcomes. Each choice of action is in effect a lottery (p_1, O_1; p_2, O_2; . . . p_n, O_n). In decision making under risk, the participant chooses between lotteries in which both the probabilities and the outcomes are explicitly given (e.g., .2, $200; .8, $0).

Expected-utility theory (Bernoulli, 1738/1954) is a normative model of decision under risk. Each outcome (O_i) in a lottery (L) is assigned a numerical "utility" $U(O_i)$, and the expected utility (EU) of the lottery is

$$EU(L) = \sum_{i=1}^{n} p_i U(O_i). \quad (1)$$

The decision maker chooses the lottery with the highest expected utility.

Human decision makers typically depart from the predictions of expected-utility theory by overweighting small probabilities and underweighting large probabilities (Allais, 1953; Kahneman & Tversky, 1979; Luce, 2000). This distortion of probability is captured by a *probability-weighting function* $W_\gamma(p)$, where γ is a free parameter, and p stands for probability. Figure 1 shows three instances of Tversky and Kahneman's (1992) probability-weighting model with one free parameter (γ). We included the curve for γ equal to 0.69 because the data observed in one condition of their experiment were best fit by a model with this value. For values of γ that are less than 1, small probabilities are overweighted, and large probabilities are underweighted. This is the typical pattern observed in experiments on decision under risk.

People are not given explicit probabilities in numerical form for most of the decisions they make. Often, people have only their own memory of the relative frequency of events across time to aid in determining probability. The relative frequency of events is arguably also the predominant source of information that nonhuman animals use to make decisions (Stephens & Krebs, 1986).

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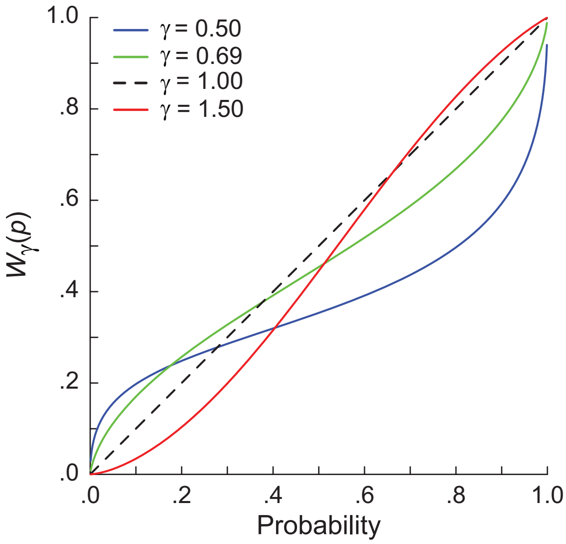

Fig. 1. Three instances of the probability-weighting function used by Tversky and Kahneman (1992). The shape of this function $W_\gamma(p)$ is controlled by a single parameter, γ (γ > 0). The function $W_\gamma(p) = p$ is a straight line when γ is equal to 1 (i.e., probability is undistorted). When γ is less than 1, the weights assigned to smaller probabilities are higher than the probabilities, and the weights assigned to larger probabilities are lower than the probabilities. The opposite pattern emerges when γ is greater than 1. The value γ = 0.69 is the actual fitted value for one condition in Tversky and Kahneman's experiments.

Recently, researchers have examined whether the source of probability information in a decision task affects the decisions people make (Erev et al., 2010; Fox & Hadar, 2006; Hadar & Fox, 2009; Hau, Pleskac, & Hertwig, 2010; Hertwig, Barron, Weber, & Erev, 2004; Rakow, Demes, & Newell, 2008; Ungemach, Chater, & Stewart, 2009). Wu, Delgado, and Maloney (2009, 2011) compared human decision making under risk in a classical decision task (viz., decision from description) and in a mathematically equivalent choice between "motor lotteries." A motor lottery consisted of a speeded reaching task to a rectangular target. If the participant hit the target, he or she earned a reward (and otherwise received nothing). The size of the rectangle determined the probability of success.

Wu and his colleagues (2009, 2011) found that although participants choosing between lotteries in a classical decision task tended to overweight small probabilities and underweight large probabilities, they showed the reverse pattern (i.e., underweighting small probabilities and overweighting large probabilities) when presented with the equivalent motor decision task. The curve in which γ is greater than 1 in Figure 1 illustrates this second pattern. Results for the same participants in a classical decision-making task with probabilities explicitly given showed the expected pattern of probability distortion (i.e., with γ < 1).

Ungemach and his colleagues (2009) reported patterned differences in the use of probability in a classical decision task and a matched decision task, in which probability information was obtained through sampling. In the classical decision task, participants chose between lotteries that were presented in numerical form. In the matched condition, participants sampled two events 40 times each. Ungemach and his colleagues found that participants underweighted smaller probabilities and overweighted larger probabilities (i.e., γ > 1, as in Fig. 1). In the classical decision task, the results showed the typical overweighting of small probabilities and underweighting of large probabilities (γ < 1). Both Wu and his colleagues (2009, 2011) and Ungemach and his colleagues (2009) verified that participants' estimates of the frequency of motor success or success in sampling were close to accurate.

A natural conjecture is that whenever probability is based on observed or remembered frequency, the use of probability in decision making will exhibit the pattern of underweighting smaller probabilities and overweighting larger probabilities (γ > 1). In the study reported here, we tested this conjecture by comparing decision making in two contexts: with probabilities learned through sampling a large number of random visual events and with probabilities explicitly stated. The random visual event was the outcome of a Gaussian "stochastic bullet" aimed at a rectangular target. The path of the stochastic bullet was a Gaussian random walk (a series of progressive steps) resembling a lightning bolt. Participants "fired" the bullet by pressing a key, but they had no influence on the bullet's trajectory. We varied the probability that the bullet would hit the rectangular target by varying the width of the rectangle. Figure 2a shows three possible trajectories of the stochastic bullet aimed at three rectangles varying in width. In the example, the bullet "hits" the rectangle two times out of three. We focused on human performance with large samples. Moreover, the number of practice and decision trials was fixed in advance by the experimenter and was not under the control of the participant (cf. Fox & Hadar, 2006; Ungemach et al., 2009).

In the visual decision task, we presented two rectangles differing in width; monetary rewards were assigned to both of these two targets. The participants choose whether to bet that the bullet would hit the upper target or the lower target. In the example shown in Figure 2b, a hit on the smaller rectangle would earn $2, but a hit on the larger rectangle, though more likely, would earn only $1. In the classical decision task, participants had to decide between two lotteries that differed in probability and outcome. In the first phase of the experiment, participants viewed 300 trials in which the stochastic bullet was aimed at rectangles of different sizes. Thus, participants could potentially learn the mapping between rectangle width and the probability that the stochastic bullet would hit the target.

We estimated probability-weighting functions in both decision-making tasks for each participant individually. Following previous researchers (Ungemach et al., 2009; Wu et al., 2009, 2011), we also asked participants to directly estimate the frequency of visual events on a numerical scale. That is, they were shown a bullet and a rectangle and were asked to estimate the probability that the bullet would hit the rectangle. We compared these three estimates of probability and frequency to test whether the source of probability information affects the probability-weighting function and whether any differences in the probability-weighting function in the two decision

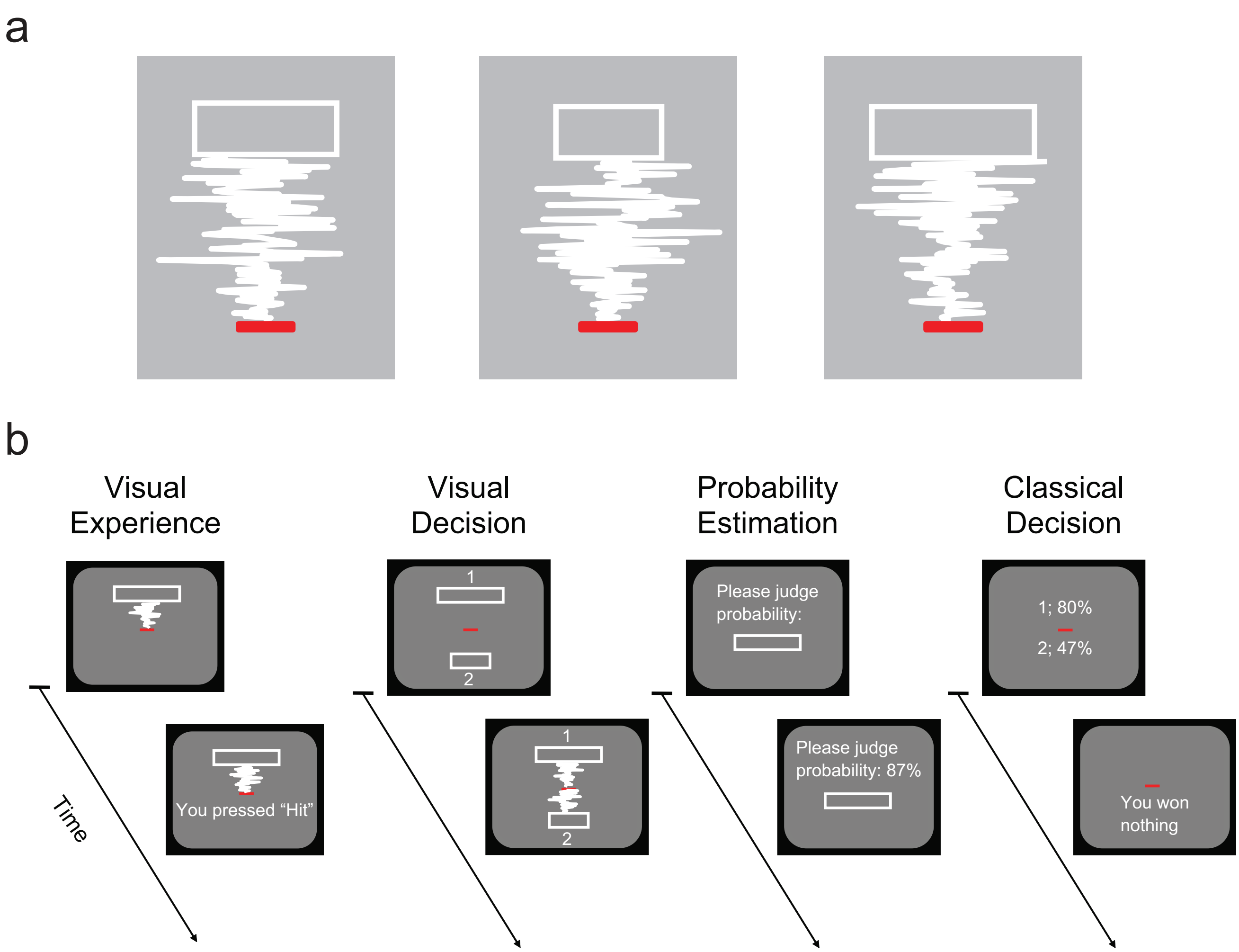

Fig. 2. Paradigm used in the visual tasks in the experiment (a) and sample trials from the first four phases of the experiment (b). Three examples of the visual stochastic process are shown in (a). A stochastic bullet was fired along a Gaussian random-walk trajectory extending from a horizontal bar known as the "shooter" toward a target rectangle. In some cases, the bullet hit the rectangle (examples in the center and on the left), and in some cases, it did not (example on the right). The path of the stochastic bullet is shown here in white; in the actual experiment, only the instantaneous positions of the moving bullet were visible. In the visual experience phase of the experiment, participants judged whether the bullet hit the target rectangle. In the visual decision phase, participants saw two target rectangles of different sizes, which were displayed above and below the shooter, respectively. Participants selected which rectangle would be shot, after which the stochastic bullet was fired twice: first toward the selected rectangle and then toward the other rectangle. If participants selected the larger target rectangle, they could potentially win $1 if it was hit; if participants selected the smaller rectangle, they could potentially win $2 if it was hit. In the probability estimation phase, participants were presented with target rectangles of different sizes, and they were asked to estimate the probability, in percentage form (between 0% and 100%), that the bullet had hit a target rectangle of that particular size in the previous phase. In the classical decision phase, participants chose between pairs of numerical lotteries, which were equivalent to the visual lotteries in the visual decision phase. Each lottery was represented as a pair of numbers: a dollar value (1 or 2) indicating the possible outcome of that lottery and the probability of that outcome, in percentage form.

tasks could be attributed to a simple misperception of the frequency of visual events.

We found large, patterned differences between the best-fitting probability-weighting functions for the two decision tasks, with the probability-weighting functions based on random visual events exhibiting greater distortion than the probability-weighting functions based on explicitly stated likelihoods. Participants typically overweighted small probabilities and underweighted large probabilities in the visual task; these patterns contrast with the patterns of probability-weight distortion found in previous research (Ungemach et al., 2009; Wu et al., 2009, 2011). We also found that participants' estimates of the relative frequency of events were close to veridical and could not explain their use of probability in visual decision making.

Method

Participants

Twelve undergraduate students of New York University participated in this experiment. All were unaware of the purpose of the experiment. All participants had normal or corrected-to-normal vision. Participants were paid for their time and also received a bonus based on their performance, which was paid at the end of the experiment.

Apparatus

Participants were seated in a dimly lit room in front of a monitor (1,280 × 1,024 pixels at 60 Hz; 1 pixel = 0.026 mm). We recorded responses with a computer keyboard mounted on a table centered in front of the monitor. The experiment was run using the Psychophysics Toolbox (Brainard, 1997; Pelli, 1997) on a Pentium IV Dell Precision workstation.

Experimental design

The experiment consisted of five phases. The first four are illustrated in Figure 2b. Participants were tested separately, and each participant completed all five phases in the following order.

Visual experience phase. The purpose of the visual experience phase was to familiarize the participant with the mapping between the width of a target rectangle and the probability that a stochastic bullet fired at the rectangle would hit the rectangle's lower edge. A single target was always displayed in the upper half of the screen, centered above the shooter. The width of the rectangle varied randomly from trial to trial.

The Gaussian stochastic bullet followed a visible random walk starting from a horizontal black line in the center of the screen (referred to as the "shooter") and proceeding upward to an invisible horizontal reward line (Fig. 2). This reward line, which coincided with the lower edge of the target rectangle, ran parallel to the shooter across the width of the display area. The random walk consisted of a series of N line segments (N = 101) connecting a series of points, beginning at the shooter and ending at the target rectangle. The coordinates of the successive points increased by $1/N$th of the distance from the shooter to the edge of the target rectangle, such that the last point fell somewhere on the lower edge of the invisible reward line. On each step of the random walk, the horizontal coordinate of the point was randomly offset from that of the previous point. The offsets were independent, drawn from a Gaussian (normal) distribution with a mean of 0 and a variance of σ_B^2/N, such that the distribution of bullet arrival points on the reward line was Gaussian (c, σ_B^2), where c denotes the horizontal coordinate of the center of the rectangle. Three realizations of the random walk are shown in Figure 2a. The distribution from which the random walk was drawn was held constant throughout the experiment, with σ_B equal to 30 mm or 3.6° of visual angle.

The target rectangle was centered directly above the shooter. The probability p of hitting a rectangle of width w with the stochastic bullet was therefore

$$p\left[\text{hit}\right] = \int_{-w/2}^{w/2} \phi\left(x;\ 0,\ \sigma_B^2\right) dx\ , \tag{2}$$

where $\phi(x;\ 0,\ \sigma_B^2)$ denotes the Gaussian probability density function with location parameter μ and variance σ_B^2, and w denotes the width of the rectangle. The mapping from w to p is invertible. Given any specified probability (p), the corresponding width of the rectangle (w) can be computed.

On each trial of the visual experience phase, the participant observed the bullet's random walk from the center of the screen upward to the lower edge of the target rectangle. The participant's task was to indicate whether or not the bullet hit the reward line in the area covered by the lower edge of the target rectangle by pressing one of two keys. Participants were told the correct response after every trial. Each participant judged 300 trials with rectangles whose widths varied randomly. The range of widths used was adjusted across 33 successive trials using an adaptive staircase procedure (Leek, 2001).

Visual decision phase. In each trial of the visual decision phase, the procedure was the same as in the visual experience phase, except that participants saw two target rectangles of differing sizes, which were displayed above and below the center of the screen, respectively. Participants selected one of these rectangles, after which a stochastic bullet was fired twice, first at the selected target rectangle and then at the nonselected target rectangle. Participants knew that they would be paid for only a small number of visual decision trials (see the description of the payoff phase later in this section). If participants selected the larger target rectangle, they could win \$1 if it was hit and if that trial were one of those selected in the payoff phase; if participants selected the smaller rectangle, they could win \$2 if it was hit and if that trial was selected in the payoff phase. If the bullet shot at their chosen target missed, a message appeared saying that the participant had won nothing. On each trial, the bullet could hit one, both, or neither of the rectangles. The larger rectangle randomly appeared above or below the shooter with equal probability.

The larger target rectangles (\$1) were fixed at one of six sizes corresponding with the probabilities of being hit (.1, .2, .4, .6, .8, .9, respectively); the size of the smaller target rectangle (\$2) was adjusted across 33 successive trials using the same adaptive staircase procedure (Leek, 2001) as in the visual experience phase. We varied the width of the target with value v_2 from trial to trial to estimate probability p_2, such that the participant was equally likely to choose (p_1, v_1) or (p_2, v_2). Each staircase terminated after a fixed number of trials. Participants completed 396 trials in the visual decision phase.

Probability estimation phase. The purpose of the probability estimation phase was to test participants' knowledge about the bullet's hit probability for target rectangles of different sizes. During this phase of the experiment, participants were presented with target rectangles of different sizes, and they estimated the probability, between 0% and 100%, that the bullet had hit the target rectangle of this particular size. The size of the target was determined by randomly selecting a probability between 10% and 90%, turning it into a width (in millimeters), and then presenting it visually to the participant. No feedback was given. This phase of the experiment consisted of 120 trials.

Classical decision phase. In the classical decision phase of the experiment, participants chose between pairs of lotteries presented in numerical form equivalent to the visual lotteries of the visual decision phase (Fig. 2b). The probability of winning the lower, \$1 outcome was fixed at the same level as in the visual decision task (.1, .2, .4, .6, .8, .9). The classical decision phase consisted of 396 trials. As in the visual task, the choice with the smaller outcome (\$1) was fixed at one of six probabilities (.1, .2, .4, .6, .8, .9) which was stated as a frequency on the screen; the probability assigned to the higher outcome (\$2) was adjusted across 33 successive trials using an adaptive staircase procedure (Leek, 2001). The values assigned to both regions remained fixed throughout the session of 396 trials. We varied the probability assigned to the higher outcome (v_1, which was equal to \$2) from trial to trial to estimate the probability p_2, such that the participant was equally like to choose (p_1, v_1) or (p_2, v_2).

Payoff phase. In the final phase, participants were paid for their participation and received an additional bonus, the sum of the outcomes of six trials: three classical lottery trials and three visual lottery trials, all selected at random. We replayed the actual random walk that had occurred on each of the three visual lottery trials, and participants received a reward for a given trial only if the rectangle they chose on that trial had been hit. These payoffs occurred only after all data were collected and the experiment was otherwise complete. Participants had been informed of this payment method at the beginning of the visual decision session.

Probability-weighting and value functions

In each phase, we fit performance to a parametric model, Kahneman and Tversky's (1979) prospect theory. The lotteries we considered had only one nonzero monetary outcome, which was always a gain. For such lotteries with two fixed values, the predictions of prospect theory coincide with those of the extension of prospect theory (cumulative prospect theory) proposed by Tversky and Kahneman (1992; Luce, 2000; Quiggin, 1982, 1993).

The participants chose one of two options: $L_1 = (p_1, v_1)$ and $L_2 = (p_2, v_2)$. We assumed that participants first assigned to each lottery a prospect value

$$\pi(L_i) = W_\gamma(p_i)V_\alpha(v_i), i = 1, 2, \tag{3}$$

where $W_\gamma(p)$ is a probability-weighting function, and $V_\alpha(v)$ is a utility function for gains. We assumed that the participant computed the decision variable as follows:

$$\Delta = \pi(L_1) - \pi(L_2) + \varepsilon, \tag{4}$$

where ε is a Gaussian random variable with a mean of 0 and a decision variance of σ_D^2. The participant chose the lottery L_1 if Δ was greater than 0 and the lottery L_2 if Δ was 0 or less than 0. Because our outcomes were nonnegative, we modeled the utility function by the power function

$$V_\alpha(v) = v^\alpha \tag{5}$$

with one parameter (α > 0). We modeled the probability-weighting function by probability as probability-weighting function with a single parameter (γ),

$$W_\gamma(p) = \frac{p^\gamma}{\left[p^\gamma + (1 - p)^\gamma\right]^{1/\gamma}}. \tag{6}$$

Examples of $W_\gamma(p)$ are shown in Figure 1.

There are competing models for how decision makers weight probability (Gonzalez & Wu, 1999; Prelec, 1998; Tversky & Kahneman, 1992), but they provide very similar fits to data (see Luce, 2000, Section 3.4). We used V_α and $W_\gamma(p)$ to capture and compare performance across experimental phases.

On every trial, there were only two nonzero outcomes: \$1 and \$2. Let v_1 denote \$1 and v_2 denote \$2. Then the comparison of lotteries,

$$W_\gamma(p_1)V_\alpha(v_1) > W_\gamma(p_2)V_\alpha(v_2) \Leftrightarrow \text{choose } L_1, \text{else } L_2, \tag{7}$$

is equivalent to

$$\frac{W_\gamma(p_1)}{W_\gamma(p_2)} > C \Leftrightarrow \text{choose } L_1, \text{else } L_2, \tag{8}$$

where $C = V_\alpha(v_2)/V_\alpha(v_1)$. The three parameters γ, C, and σ_D characterized each participant's performance.

Analysis and Results

In each of the decision phases and for each participant, we estimated parameters by the method of maximum likelihood (Mood, Graybill, & Boes, 1974). We combined data across the six probability conditions (.1, .2, .4, .6, .8, .9), a total of 396 trials.

The resulting estimates of γ, C, and σ_D for the visual decision task are denoted γ_k^v, C_k^v, and $\sigma_{D,k}^v$, respectively, with k varying from 1 to 12 to index the 12 participants. We also obtained the maximum log likelihood of each fit, λ_d^v, which we used when testing hypotheses. We fit the data for the classical decision phase similarly, obtaining estimates of γ, C, and σ_D denoted γ_k^c, C_k^c, and $\sigma_{D,k}^c$, with k equal to 1 through 12 and corresponding maximum log likelihoods, λ_k^c.

Finally, we fit the data from the estimation phase by the method of least squares to obtain estimates γ_k^e with k equal to 1 through 12, as well as the estimated standard error of the residuals, σ_k^e. The data are estimates $E_{k,j}$ of probability (on a percentage scale) by the kth participant on the jth trial of the estimation task. The true probability that the bullet would hit the target on the jth trial of the estimation task is denoted p_j. We fit the parameter γ_k^e by minimizing,

$$\sum_{j=1}^{N_E}\left[E_{k,j} - 100\beta_k\, w_{\gamma_k^e}\left(p_j\right)\right]^2. \tag{9}$$

Note that we fit an additional parameter β_k that captured any tendency on the participant's part to compress or expand the response scale.

Distortion of probability-weighting functions

In all three conditions, we found large individual differences in the γ estimates; this indicated that the extent to which probability information was distorted varied across participants. In the visual task, individual γ estimates varied between 0.31 and 1 (median = 0.725). In the classical decision task, individual γ estimates ranged between 0.28 and 2 (median = 1.05). In the estimation task, we found large individual differences in γ estimates, with values across participants that ranged from 0.36 to 1.55 (median = 1.08).

Visual and classical decision tasks. The estimates for γ_k^c, γ_k^v, and γ_k^e for each of the 12 participants (k = 1–12) are summarized in Figure 3a. The large individual differences for the classical decision task are typical of those found when participants' data are fit individually (Luce, 2000, Section 3.4). The mean of 1.01 (median = 1.05) was not significantly different from 1, $t(11) = 0.103$, $p = .459$; the individual values were evenly distributed around 1. In contrast, all values of γ_k^v were less than 1, with a mean value of 0.721 (median = 0.725). This difference is significant, $t(11) = -4.323$, $p = .0012$. Figure 3b shows box plots indicating mean and confidence intervals for the visual and classical decision tasks, with parameter estimates pooled across the 12 participants specifying the probability-weighting function. The parameter estimates differed significantly for the visual and classical decision task for 8 of the 12 participants (log-likelihood-ratio test, $p = .05$).

We believe that the lack of probability weighting (on average) in the classical decision session was due to the feedback that participants received on each trial. People faced with a series of decisions tend to move closer to maximum expected gain (see Redelmeier & Tversky, 1992; Thaler & Johnson, 1990; Wakker, Thaler, & Tversky, 1997). Studies of risky decision have found that participants are closer to maximizing expected gain for small stakes (Camerer, 1992; Holt & Laury, 2002; Thaler & Johnson, 1990; Wedell & Böckenholt, 1994).

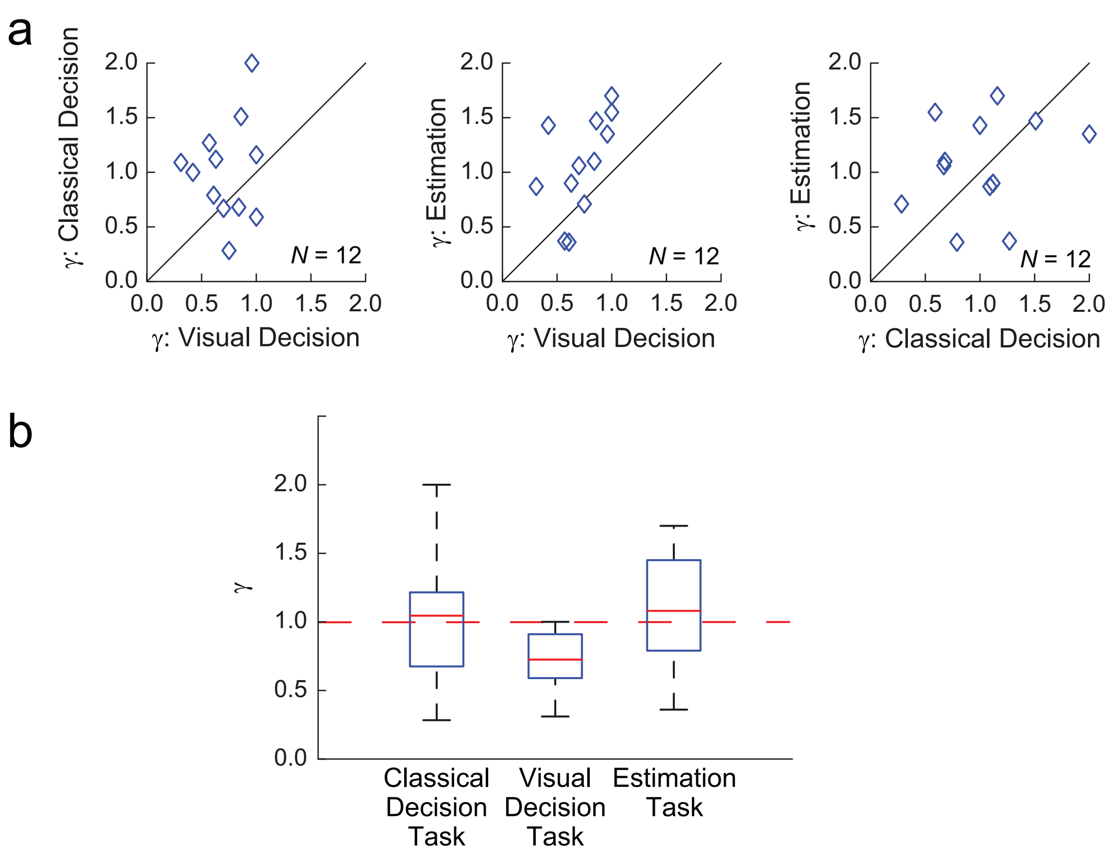

Fig. 3. Values of the parameter estimates (γ) specifying the probability-weighting function in the classical decision, visual decision, and estimation tasks. The scatter plots in (a) show the relation between the parameter estimates in each pair of tasks. The box-and-whisker plots in (b) show the medians (solid red lines) of γ in the same three phases of the experiment. The dashed vertical lines show the full range of γ values. The lower and upper edges of the blue rectangles mark the lowest and highest quartiles.

Similar results have been found for motor tasks analogous to decision making (Trommershäuser, Maloney, & Landy, 2003a, 2003b, 2008).

We also tested whether γ^v and γ^c covary. Do participants with larger γ^c values tend to have larger γ^v values, and vice versa (positive correlation between γ^c and γ^v)? We computed the correlation between γ_k^v and γ_k^c across participants. We found that participants did not show a significant correlation between visual decision (γ^v) and classical decision (γ^c; $r = -.17$, $p = .61$). We thus conclude that probability-weight functions in the two decision tasks were different across participants, with the probability-weight function for the visual task more distorted.

Estimation task. We found large individual differences in γ_k^e, with values across participants ranging from 0.36 to 1.55 ($M = 1.07$, median = 1.08). Estimates of the scaling parameter β_k ranged from 0.63 to 2.3 ($M = 1.07$, median = .94). The values of γ_k^e from the estimation task were not significantly different from 1 ($M = 1.07$, median = 1.08), $t(11) = 0.563$, $p = .585$. On average, participants' estimates of the frequency that the bullet would hit the target were not significantly distorted.

Probability estimates and the visual decision task. We considered whether the observed distortions in probability weight could be due to misperception of the frequency with which the stochastic bullet hit targets of different widths. We first tested the hypothesis that $\gamma^v = \gamma^e$, and results showed that this hypothesis was not valid, $t(11) = -3.256$, $p = .008$. We further tested whether γ^v was positively correlated with γ^e. The resulting correlation was only marginally significantly different from 0 ($r = .55$, $p = .066$). This outcome is consistent with the findings of previous work comparing estimates and the use of probability in decisions under risk or uncertainty (Ungemach et al., 2009; Wu et al., 2009, 2011).

Distortion of value functions

The cumulative-prospect-theory value function for gains is controlled by a single parameter (α), which we reduced to a single parameter (C). The estimates for each participant in the visual and classical decision tasks are denoted C_k^v ($M = 1.338$, median = 1.230) and C_k^c ($M = 1.425$, median = 1.255). The ratio of rewards was 2:1 and a value of C equal to 2 would indicate that participants made decisions consistent with expected value. We tested and rejected this possibility for C_k^v, $t(11) = -7.1832$, $p < .0001$, and for C_k^c, $t(11) = -3.367$, $p = .006$. In both decision conditions, our estimates of participants' use of value were compressive (with slope decreasing as magnitude increases), as is typically the case for lotteries with nonnegative outcomes (Tversky & Kahneman, 1992).

We finally tested whether participants treated information about reward similarly in the two decision conditions. We compared C_k^v and C_k^c using a paired-samples t test and found no significant difference, $t(11) = -0.773$, $p = .456$. Moreover, C_k^v and C_k^c were significantly correlated, $r = .789$, $p = .002$. These results are consistent with the conclusion that participants represented value similarly in the two decision conditions.

Conclusion

Previous researchers (Ungemach et al., 2009; Wu et al., 2009, 2011) compared human performance in classical decision making (in which information about probability and outcomes was explicitly given) to performance in a matched decision task (in which knowledge of probability was obtained by observing or sampling stochastic events). These researchers found that decision makers in the classical task typically overweighted small probabilities and underweighted large probabilities. In contrast, they found the opposite pattern for the matched task based on relative frequency, which suggests that the shape of the probability-weighting function changes markedly when probability is obtained in the form of relative frequency of events.

We tested this conjecture by comparing performance in a classical decision task with performance in a visual decision task. Participants practiced firing bullets at rectangles several hundred times. In this visual decision task, the participant chose between firing a bullet at a larger, easier-to-hit rectangle for $1 or a smaller, harder-to-hit rectangle for $2. We varied the widths of the rectangles from trial to trial; the objective probabilities of success in the visual task and the classical task were matched on a trial-by-trial basis.

Our results showed that the shape of the probability-weighting function does not change markedly when probability is obtained in the form of relative frequencies. In the visual task, the values for γ^v were all less than 1, indicating that participants typically overweighted small probabilities and underweighted large probabilities. This pattern is the opposite of that found by Ungemach and his colleagues (2009) and Wu and his colleagues (2009, 2011), even after 300 practice trials and 396 decision trials in the visual decision phase. Sample size alone is not sufficient to account for differences in human choice in decisions from description and decisions from sampling.

In summary, we compared performance in a decision-from-description task and a decision-from-sampling task, in which the source of uncertainty was a stochastic visual event. We found that even after many trials, participants' probability-weighting functions for description and for sampling were significantly different, and the probability-weighting functions from sampling showed marked overweighting of small probabilities and underweighting of large probabilities. Even when we estimated probabilities as the frequency of large numbers of visual events, we did not find that observers maximized expected utility.

The task we considered was representative of a large class of everyday tasks, in which one learns about the stochastic outcomes of events by observing, a quintessential form of decision from experience. It might be expected that with hundreds of trials of training and decision, human performance would move toward choices maximizing expected utility. However, we found that it does not.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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