The influence function for visual interpolation

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ABSTRACT

We present three experiments in which observers were asked to interpolate parabolic sampled contours. In the first two experiments, observers saw only eight isolated sample points, all of which lay on or near an otherwise invisible contour. The observer adjusted the position of a ninth point until s/he judged it to be on the contour as well. We measured the effect of small perturbations in the locations of each of the visible points on the observer's setting and derived a locally linear "receptive field" that characterizes how each of the visible points contributed to the interpolation judgment. In the third experiment, we develop a measure of segmentation performance based on the same methods and analyses.

Keywords: interpolation, segmentation

1. INTRODUCTION

We present experiments concerning human perception of contour in simple scenes made up of identical, isolated points presented against an otherwise uniform background. The initial sensory information available to the observer in such a scene is easily summarized: we need only describe the physical characteristics of any one of the points and then list their (x, y) coordinates. The observer's perceptual experience in viewing such a scene is less readily summarized.

We will use the term *sampled contour* in referring to such an (invisible) contour sketched by points (e.g., Fig. 1a). We do not intend to suggest that observers see a continuous contour connecting the points in Fig. 1a. In that respect, a sampled contour is unlike a physical contour or a "subjective" contour.[1] *Some* sampled contours, including the one in Fig. 1a, share a second property of physical contours: they can be precisely and objectively *interpolated* .

In a series of experiments, Koh and Maloney[2,3] asked observers to interpolate points on parabolic sampled contours such as the one shown in Fig. 1a (we use the same interpolation task as they did and we will describe it in detail below). They found that, given as few as 5-7 points, observers could accurately interpolate contours. Different observers made almost identical settings. Their interpolated points lay very close to the unique, invisible, parabolic contour containing the points that the experimenters had selected.

By varying the number of visible points or the choice of contour, it is easy to destroy this objective agreement among observers. Observers are very unlikely agree with each other or with the experimenters' choice of sampled contour in Fig. 1b. We've sketched in the "invisible" contour as it would be otherwise difficult to infer.

In this paper we are interested in studying the kind of objectively successful interpolation that observers seem able to perform on some sampled contours including the one in Fig. 1a. We develop a quantitative measure of the *influence* of each visible sample point on the observer's interpolation of an invisible sampled contour. We describe this influence measure below. In two experiments, we use it to explore how human interpolation performance depends on each point in the sampled contour.

Of course, a scene may contain no sampled contours, exactly one, or several. A human observer can readily *segment* many scenes containing multiple overlapping sampled contours. This *segmentation problem* is known to be computationally intractable[4] and it would be of interest to know how human observers go about solving it in the cases where they seem able to do so. How do they decide which point goes with which contour? In the third

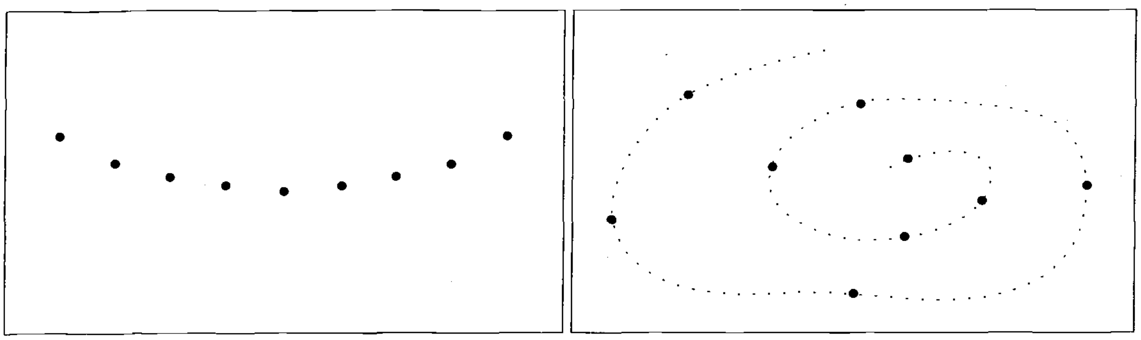

Figure 1. (A) A set of points that fall on a parabolic contour. Observers find this contour easy to interpolate. (B) A set of points on a smooth contour. Observers are unlikely to infer the indicated smooth contour (or any other single, smooth contour) from the sample points.

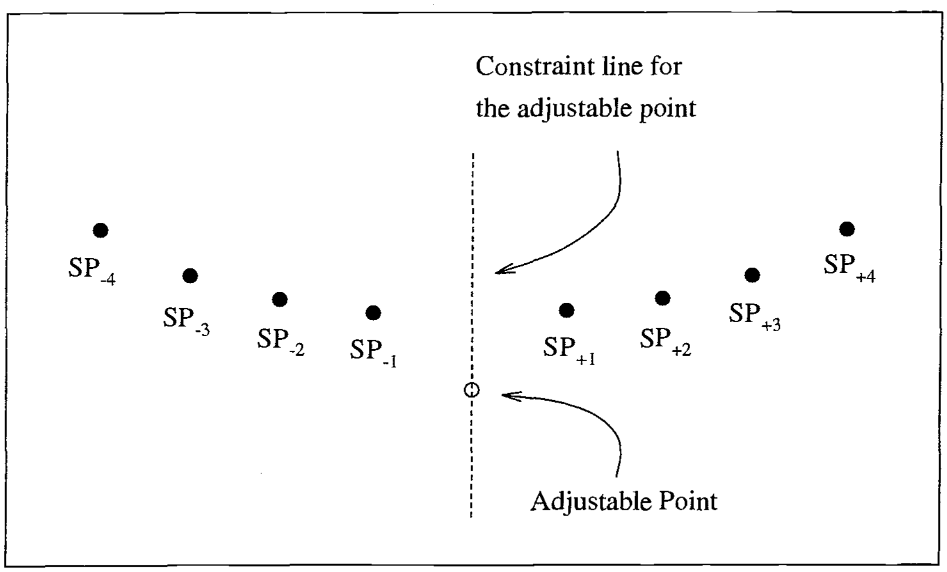

Figure 2. This is a general description of the stimuli used in the experiments presented here. Nine points are sampled from a parabolic contour. The observer adjusts the center adjustable point (AP) so as to appear to lie on the perceived contour suggested by the other eight sample points (SPs) some of which were systematically perturbed from their nominally correct positions on the intended contour. We measured the influence of those eight SPs on the perception of the contour by comparing the changes in the setting of the AP relative to the size of the perturbation.

experiment reported, we use the same influence measure to explore how observers segment a scene containing a single sampled contour and additional isolated points, one of which may be close to the contour.

Observers were presented with nine points sampled from a parabolic contour and were instructed to move the center point, hereafter referred to as the *adjustable point* or AP, along an invisible constraint line until it appeared to lie on the perceived contour containing the other eight points (Fig. 2). The eight remaining points, called the *sampled points*, are designated SP_{-4}, SP_{-3}, SP_{-2}, SP_{-1}, SP_{+1}, SP_{+2}, SP_{+3}, and SP_{+4}. The subscript denotes the

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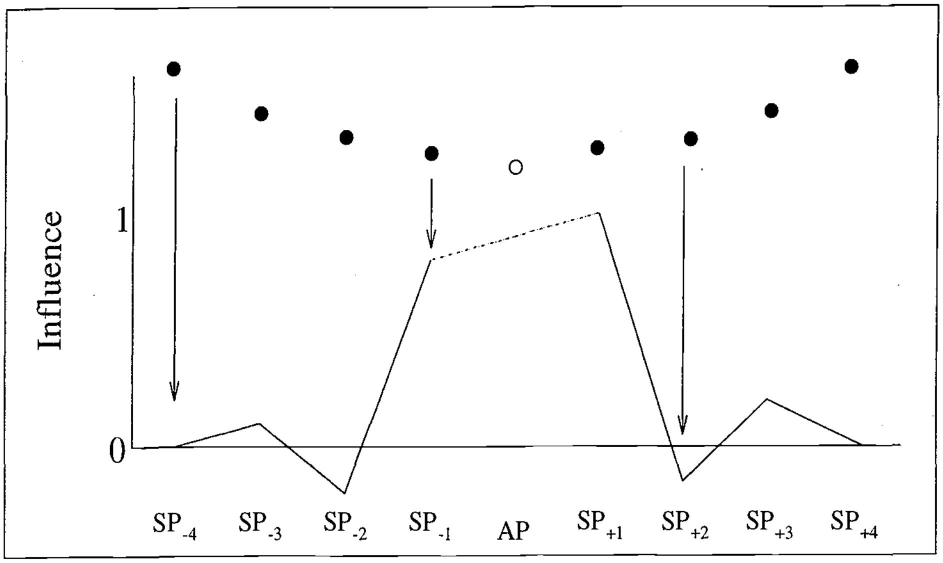

Figure 3. A hypothetical plot of influence. The ordinate is the measure of influence relative to the size and direction of the perturbation. A positive influence means that the change in the setting is in the same direction as the perturbation. A negative influence means that the change in the setting goes in the opposite direction of the perturbation.

position of each point along the contour relative to the AP. The AP can be thought of as SP_0.

We wish to quantify the contribution of each sample point to the observer's setting. To do so we use a *perturbation method* similar to that discussed by Landy, Maloney, Johnston and Young.[5]

Influence. In this set of experiments, a perturbation refers to a change in the location of a SP from its nominally correct location. On trials without perturbed points, the stimulus presented to the observer shows the SPs on the parabolic contour. On a perturbation trial, one or more of the SPs is shown slightly displaced from its originally sampled location.

A perturbation can be represented by a vector that specifies the direction and magnitude of displacement. In this paper we will consider perturbations along only one direction, parallel to the constraint line containing the AP (the vertical direction in Fig. 2). The magnitude of the perturbation along this line is denoted p and is a signed quantity. We did not measure perturbations in other directions because of the shallow curvature of the parabolic contours employed. Small horizontal displacements of points in Fig. 2 would tend to move them along the contour rather than away from it, while altering the approximately uniform spacing between points. In these initial studies of the use of perturbation, we chose to avoid these potential complications.

The *effect* of the perturbation, $E_i(p)$, is defined to be the difference in the observer's settings of the AP when the SP_i is perturbed and when it is not perturbed. Letting $A_i(p)$ denote the position of the setting along the constraint line when SP_i is perturbed by p, then

$$E_i(p) = A_i(p) - A_i(0). \tag{1}$$

The effect, of course, depends on the magnitude of the perturbation. For sufficiently small perturbation magnitudes p, we might expect that the ratio

$$I_i = E_i(p)/p \tag{2}$$

would be constant and independent of p. The value I_i is called the *influence* of the perturbation. We will test whether it is independent of p for small p in Exp. I.

Fig. 3 depicts a hypothetical plot of the measurement of influence along a sampled contour. The abscissa refers to the relative positions of the SPs. The ordinate is the influence that SP_i has on the AP. For example, perturbing point SP_{-4} has no influence on the setting of the AP. Point SP_{-1}, however, has a strong influence on the location of the AP. In addition, the fact that the influence is positive tells us that perturbing point SP_{-1} pulls the AP in the same direction as the perturbation. Point SP_{+2} also has an influence on the setting of the AP, but the influence is weak and is negative. The negative value of the influence indicates that perturbing this point will change the AP setting in the opposite direction of the perturbation.

2. GENERAL METHODS

2.1. Stimuli

All experiments were performed using an Electrohome ECP 4100 series projection monitor driven by a Cambridge Research System VSG 2/3 graphics board. The observers viewed the projected stimuli at an approximate distance of 5.5 m. From this distance the screen subtends about 7×9 deg. The resolution of the screen is 896 horizontal by 566 vertical pixels. This aspect ratio was selected because it yielded square pixels.

2.2. Calibration

The positions of points projected onto the display area are not precisely a linear function of the internal xy graphics coordinates of the CRS board. We corrected this spatial miscalibration as follows. Before each experimental session, the observer ran a spatial calibration program. Using this program, s/he would position a displayed point at each of the 81 intersections of a 9×9 rectangular grid faintly marked on the display area (this grid was not visible to the observer during the experiments). The calibration program recorded the graphics coordinates corresponding to the 81 physical reference points. This information was then used by experimental programs to interpolate the graphics coordinates needed to display a point at any desired physical location in the display area using interpolation routines taken from *Numerical Recipes*.[6] This method of geometric calibration was checked before any experimental trials were run. It was found to be accurate to within half a pixel, the maximum achievable without anti-aliasing.

3. EXPERIMENTS

3.1. Experiment I

In the first experiment, we ask "How does each SP_i affect the observer's setting of the AP?" or, put in another way, "What is the *influence* of each point?" We also test whether the measured influence of each SP_i depends on the overall scale of the sampled contour.

3.1.1. Observers

Four observers participated in this experiment. Two of the observers were the authors. The other two were naive, paid observers.

3.1.2. Methods

Two different fixed parabolic contour segments were used. They were both asymmetric about the AP and one had a slightly higher curvature than the other.

The positions of the eight sampled points SP were chosen as follows. First, the total length of the contour was computed by numerical contour integration. Then, this length L was divided by eight to give the mean interpoint spacing $S = L/8$. The mean unperturbed position of point SP_i ($i = \pm 4, \pm 3, \pm 2, \pm 1, 0$) was $(i+4)S + U$ measured along the contour. U was a random uniform perturbation with mean 0 and standard deviation 0.1 of the average interpoint spacing. The result of these computations was nine approximately equally spaced points all falling exactly on the contour with the first and last points near its two ends. The point SP_0 (corresponding to the nominally correct position of the AP) was never displayed. The remaining eight points were the displayed SPs.

Rotation and Scaling. The entire stimulus array, including all constraint lines and perturbations, was randomly rotated by 0, 45 or 90 deg. Each stimulus was either displayed as is or reflected about the vertical axis, chosen randomly on each trial. The entire stimulus array was displayed at one of two scales. In the larger scale, the two sampled parabolic contours (both of which resembled Fig. 2) fit into rectangles measuring about 7.10×1.25 deg and 6.22×2.22 deg, respectively.

Points. Inter-pixel spacing was 1 min. Each displayed point was a 4×4 square of pixels, or 4 min on a side. The spacing between adjacent *SP*s measured along the contour was approximately 1 deg at the larger scale. The *AP* was initially displaced to a random location on an invisible constraint line (Fig. 2).

Perturbation. On a given trial, any one *SP* could be perturbed. The perturbation could be one of two magnitudes: 4 or 8 min for the large scale, and 2 or 4 min for the small scale. The perturbations were constrained to lie on a line that was parallel to the *AP*s constraint line (see Fig. 2 and below) and passed through the perturbed point's originally sampled location. The perturbation could be positive, negative or zero. If positive, then the point was displaced toward the concave side of the parabola. If negative, then it was displaced toward the convex side. A perturbation of zero corresponded to the originally sampled location.

Feedback. On trials where there was no perturbation, observers received feedback as to how close they were to the true parabolic contour. This was done by briefly flashing the nominally correct point and a point along the constraint line that was ten times further away from the true point than, and in the same direction as, the observer's setting. This magnification was necessary because the true point and the observer's setting were usually too close to be distinguished.

Sessions. There were 216 possible combinations of the factors above: 3 rotations, 2 scales, 2 magnitudes of perturbation, 2 contours, and 9 perturbation conditions (including the no perturbation condition). Each session included all the conditions presented in a random order. Each observer completed 6 sessions.

3.1.3. Results

Fig. 4 shows the data for the four observers. The plots are broken down by condition. The upper left hand plot shows that data for the large scale stimulus perturbed by large perturbations. The lower right hand plots shows the data for the small scale stimulus under the small perturbation condition. The other two plots show the remaining two conditions. Each data point is an average of 36 settings (3 rotations, 2 contours, 6 sessions).

In general, the data indicate that the two points adjacent to the *AP*, SP_{-1} and SP_{+1} exert more positive influence than other points. Although, for most of the observers, there is an asymmetry between the influence of the point to the left of the *AP* and the point to the right, the influence is never in the opposite direction to the perturbation. The maximum influence of these two points is about 1.0. The remaining six points exhibit a less consistent pattern. Generally, somewhere around $SP_{\pm 2}$ and $SP_{\pm 3}$ perturbing the *SP*s has a negative effect on the *AP*. For most of the observers, under most of the conditions, the point furthest from the *AP* exerts essentially no influence.

We performed an ANOVA on the measured influence and found that AKH and LTM were not rotationally invariant. We also found that NXD was not scale invariant. However, the deviations, though statistically significant, were very small in magnitude. For all observers, the magnitude of the perturbation did not alter measured influence. This is consistent with the claim that influence is independent of the precise magnitude of perturbation for sufficiently small magnitudes.

3.2. Experiment II

It is tempting to interpret the perturbation curves of Fig. 4 as a locally linear approximation to the human visual interpolation function, that is, as a "receptive field." In Exp. I, we implicitly tested *homogeneity*: doubling the magnitude of perturbation led to a doubled effect and, consequently, no change in influence. Before we can claim that the perturbation graphs can be interpreted as a locally linear receptive field, we will test superposition. Is the effect of perturbing two sample points the sum of the effects of perturbing each in isolation? We test this assumption next.

3.2.1. Observers

The observers used in this study were two of the authors.

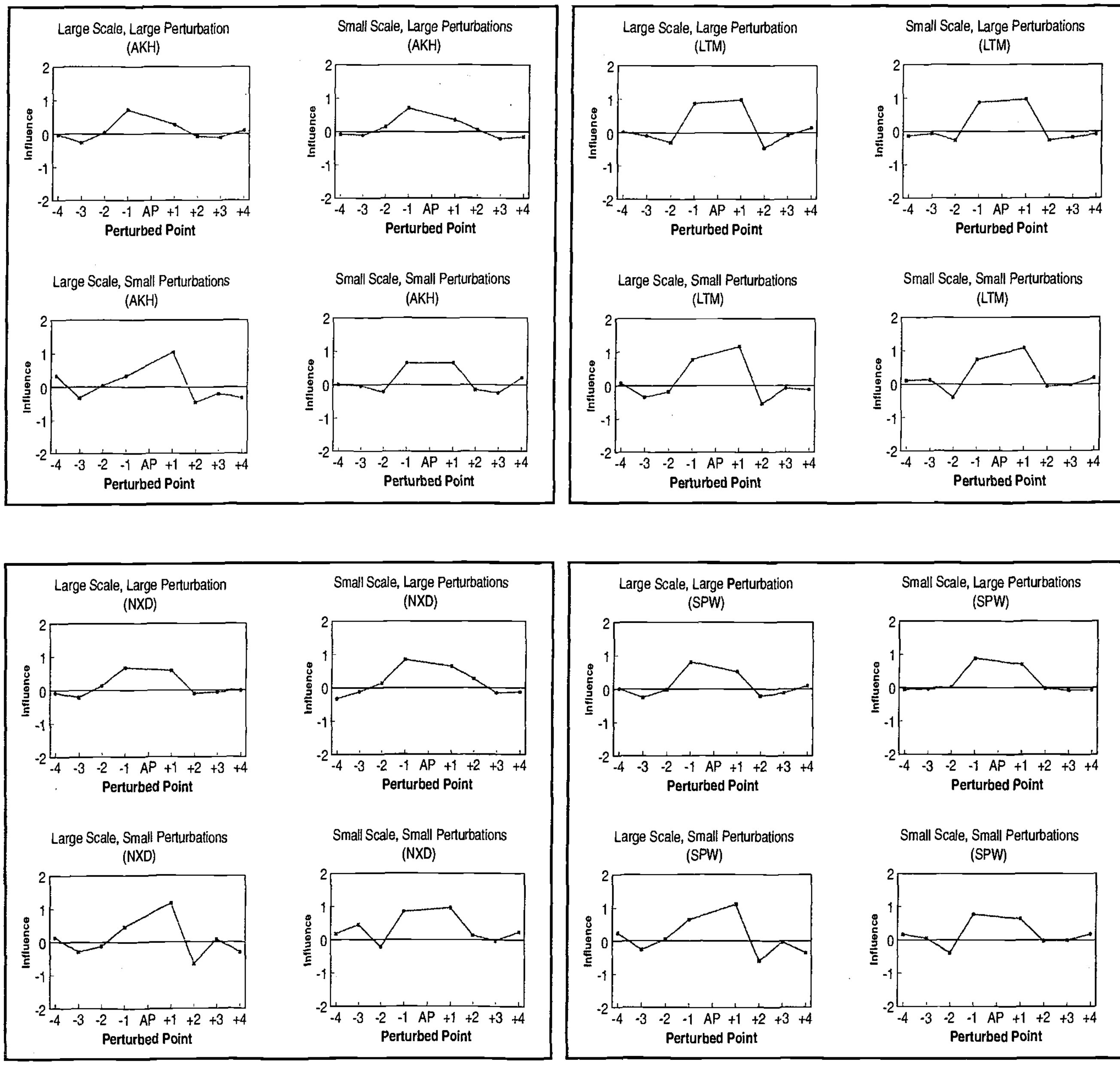

Figure 4. Data from Exp. I. Each plot shows the influence that the SPs have on the AP under the four conditions for each of the four observers.

3.2.2. Methods

The general methodology of this experiment follows that of Exp. I. Only one parabolic contour was used in this experiment. Since differences in scale did not produce physically significant results in Exp. I, stimuli were presented at only the larger scale. Only points SP_{-2}, SP_{-1}, SP_{+1}, and SP_{+2} were perturbed (in both directions). In each trial, either none of the points were perturbed, one of the four points was perturbed (as in Exp. I) or two of the points were perturbed at the same time. All of the perturbations were 4 min.

There were six possible ways of pairing the four points: (SP_{-2}, SP_{-1}), (SP_{-2}, SP_{+1}), (SP_{-2}, SP_{+2}), (SP_{-1}, SP_{+1}), (SP_{-1}, SP_{+2}), and (SP_{+1}, SP_{+2}). For each pairwise perturbation, there were 4 directional pairs: positive-positive, positive-negative, negative-positive and negative-negative. An example of one of the paired perturbation conditions would be: perturb SP_{-2} in the positive direction and SP_{+1} in the negative direction. Altogether, there were 24 such conditions.

Each observer ran 4 sessions of 102 trials consisting of all the combinations of the conditions: 24 paired perturbations trials, 8 single perturbations trials (4 points x 2 directions) and 2 unperturbed trials, all multiplied by 3 angles of rotation. Again, stimuli were randomly reflected about a vertical axis.

3.2.3. Results

We compared the effect of each pairwise perturbation to the sum of the effects of the corresponding single perturbations and tested for equality by a t-test at the 0.01 level. None of the t-values were found to be significant. We could not reject the hypothesis that observers' interpolation performance obeyed superposition.

The results so far suggest that the influence measure allows us to characterize the role of each visible sampled point in determining the observer's perception of the contour, as measured by the interpolation task. The results of Exps. I and II indicate that we can measure a locally linear operator which we refer to as an *influence field* that predicts observer's interpolation performance with multiple, small perturbations of the SPs. Fig. 4 shows empirical estimates of influence fields.

3.3. Experiment III

In the first two experiments, the SPs were always on or near the parabolic contour. In this third experiment, we wish to use influence measures to characterize whether a given point is perceived as part of a sampled contour or not. In this experiment, we gradually move a contour point away from the contour and measure the change in the effect of the point on interpolation at a fixed location along the contour. We make the following linking hypothesis concerning influence and membership in a contour: If a point has no influence on interpolation performance at any point along a contour, it is not part of the contour. Conversely, if the point has influence for interpolation performance at at least some points along the contour, then it belongs to the contour.

We are especially interested in seeing what happens as the displaced point is moved farther away from the contour. Does it suddenly cease to belong to the contour in an all-or-none fashion? Or, does it gradually lose membership (influence)? We will return to these points in the Discussion section.

3.3.1. Observers

The observers in Exp. III are the same as those in Exp. I.

3.3.2. Methods

The SPs were sampled from only one parabolic contour. The stimulus array was presented at only one scale which was identical to the large scale in Exp. I. The entire stimulus array, including all constraint lines, was randomly rotated and reflected as in Exp. I. On any given trial, any one SP could be perturbed. The perturbation could be one of five magnitudes: 5, 8, 11, 22 or 44 min. For comparison, the large perturbation in Exp. I was 8 min and the interpoint spacing was about 1 deg. The perturbation could be either in the positive or the negative direction. Each observer ran 10 sessions of 150 trials each.

Along with the SPs, five randomly placed noise points were presented. They served as "background", secondary "objects", intended to encourage the observer to exclude the perturbed point from the contour. The noise points were constrained to be at least 1 deg from any SP. As noted above, the maximum displacement of the perturbed point was 44 min.

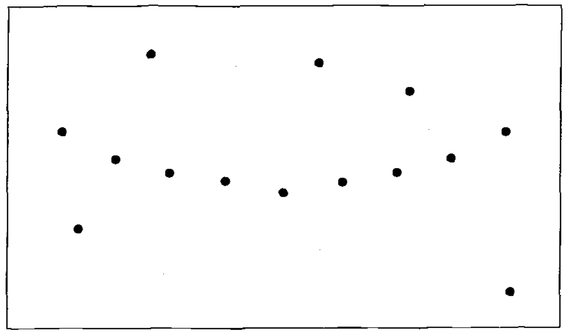

Figure 5. Example of a stimulus surrounded by five "noise" points.

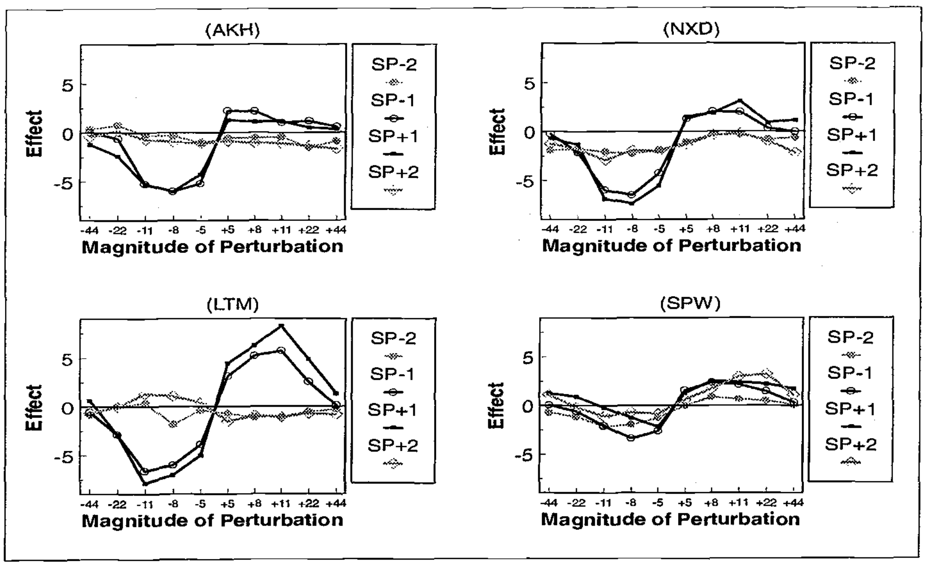

Figure 6. Data from Exp. III. When the perturbation is small, $SP_{\pm 1}$ have strong effects on the setting. However, as the magnitude of perturbation increases, the magnitude of effect on the AP reaches a maximum and starts to decline. Eventually, when the point is perturbed far enough, it exerts no effect on the setting of the AP (e.g., is no longer perceived as belonging to the contour). All units are in minutes of arc.

3.3.3. Results

Fig. 6 shows the results of Exp. III. On the abscissa is the magnitude and direction of perturbation. The ordinate is the magnitude and direction of the effect. For the two points adjacent to the AP, SP_{-1} and SP_{+1}, shown with the dark lines, the curve shows a maximum magnitude of effect for perturbations of approximately 8 min, and a dropoff after this point. For observers AKH and NXD, there is also a sizable asymmetry in the effects. Apparently, for these two observers, the effect of the perturbation is less when the SP was displaced toward the concave side of the parabola. For the points SP_{-2} and SP_{+2}, the qualitative effects are similar, but the magnitude is smaller.

At small perturbations, as before, the effect of the perturbation grows linearly with the perturbation and the influence is, therefore, constant. At larger perturbations, the effect continues to increase but more slowly than the perturbation. For larger perturbations, the effect begins to *decrease* with increasing perturbation, and when the perturbation is sufficiently large, the displaced point has no effect on the perceived shape of the contour.

3.3.4. Discussion

The smooth shape of Fig. 6 is consistent with at least two competing interpretations. We can interpret the reduction in influence of the displaced point as a measure of partial *membership* of the point in the contour. We hypothesize a membership weight or measure $M(p)$ between 0 and 1 that is assigned to each point p. When $M(p)$ is 0, the point p is not a member of the contour and should not have any effect on interpolation performance. When $M(p)$ is 1, the point p is a member of the contour. When $M(p)$ is between 0 and 1, the influence of the point on interpolation is attenuated: the point is in effect treated as "suspicious". Under this *Graded Membership Hypothesis*, the notion of "membership" in the contour is a graded, continuous concept. This sort of down-weighting of "suspicious" points or outliers is common in robust statistical computations.[7]

Alternatively, we can imagine that a point, on any single trial, is either a "full" member of a sampled contour, or not. We will refer to this hypothesis as the *All-or-None Hypothesis.* For any fixed degree of perturbation, the observer can either (a) include the perturbed point as part of the contour and compute the interpolation setting accordingly, or (b) exclude the perturbed point from the contour and base the interpolation judgment on the remaining points. We denote the setting on "included trials" by $A_i(p)$ and the setting on "excluded trials" by A_e. The latter, of course, does not depend on the magnitude of perturbation p. The probability of inclusion on any given trial is denoted $\pi(p)$. The measured effect $A(p)$, averaged across trials, is then a mixture,

$$A(p) = \pi(p)A_i(p) + (1 - \pi(p))A_e \tag{3}$$

of the setting on trials where the perturbed point is included or excluded. There are many choices of $\pi(p)$ and $A_i(p)$ that lead to the curve of Fig. 6. For example, $A_i(p)$ could be linear, continuing the line observed with small perturbations. We can then choose $\pi(p)$ to produce exactly the results of the experiment.

Both the Graded Membership Hypothesis and the All-or-None Hypothesis can account for the mean settings as a function of perturbation magnitude. Since the All-or-None Hypothesis implies that observers' mean settings in Fig. 6 are mixtures of two settings across trials, it is natural to consider how the variance $V(p)$ of observers' settings changes with perturbation magnitude. We develop the All-or-None Hypothesis further by assuming that the inclusion probability π decreases from 1 to 0 as the absolute value of the magnitude of perturbation p goes from 0 to ∞. When the perturbed point falls on the parabolic contour it is always included and at some sufficiently large distance from the contour it is always excluded.

Let $V_i(p)$ denote the observer's intrinsic variance in setting on any single trial for trials in which the perturbed point was included, and V_e for trials in which it was excluded. As the notation indicates, $V_i(p)$ may depend on the magnitude of perturbation p, and obviously V_e does not. Under the All-or-None Hypothesis, the distribution of responses is a mixture of two underlying distributions (for "included" and "excluded" trials). A simple computation shows that the variance in setting observed across trials, $V(p)$, is the sum of the observer's intrinsic variance and a *mixture term* that depends on the settings on included and excluded trials as well as the probability of inclusion:

$$V(p) = \pi(p)V_i(p) + (1 - \pi(p))V_e + [\pi(p)(A_i(p)^2 - A(p)^2) + (1 - \pi(p))(A_e^2 - A(p)^2)]. \tag{4}$$

The mixture term (in square brackets) is always positive.

We further assume that $V(p)$ is constant or nearly so, and equal to V_e, an assumption we will justify in the sequel. With this last assumption, when π is 0 or 1, the observed variance will be the intrinsic variance. Thus, the plot of variance $V(p)$ versus magnitude of perturbation p under the All-or-None Hypothesis is predicted to be the sum of a constant, intrinsic variance and the mixture term in the equation above: variance will initially increase with increasing absolute value of perturbation magnitude and then decrease to its initial level (as the mixture term returns to 0, that is, as π approaches 0). We expect a plot that is qualitatively shaped like the letter "M".

Under the Graded Membership Hypothesis, we might expect $V(p)$ to increase, to decrease, or to be constant. We can find no plausible set of assumptions that lead to the same M-shaped pattern predicted by the All-or-None Hypothesis.

In Fig. 7 we plot the standard deviation $SD(p)$ (the square root of $V(p)$) of observers' settings versus magnitude of perturbation for two of the perturbed points: SP_{-1} (black bars) and SP_{+1} (white bars). Note first that $SD(p)$ for $p = 0$ is little different from $SD(p)$ for $p = 44$ and $p = -44$ min (the extremes), consistent with the assumption that intrinsic variance is constant or nearly so. Three of the four observers show an increased SD for intermediate values of perturbation in both positive and negative directions (the M-shaped pattern). The remaining observer (SPW) shows the pattern for SP_{+1} but not SP_{-1}.

We certainly cannot reject the All-or-None Hypothesis, given the data, but we do not consider the evidence in its favor particularly strong. Further experiments, specifically designed to test for bimodality in the observer's settings, are needed to resolve whether the All-or-None Hypothesis correctly describes observers' performance.

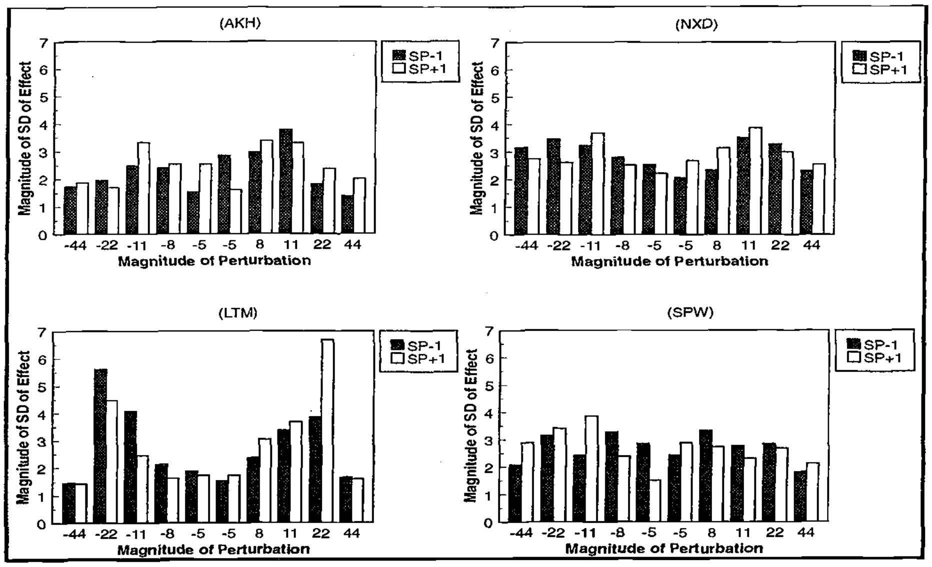

Figure 7. Standard deviations of the data from Exp. III.

4. SUMMARY

In Exps. I and II, we measured the influence of each of eight sample points on observers' interpolation of parabolic sampled contours. The influence measure is a local linearization of a network of constraints the visible points make on the observer's perception of the invisible, interpolated contour. The results of Exps. I and II indicate that the combined influence measures can indeed be thought of as a linear receptive field, summarizing the effect of small displacements of one or more points.

These *influence fields* are presented in Fig. 4. They are scale-invariant for three of four observers and approximately so for the remaining observer. They are remarkably local. Although the observer has eight visible points to use in interpolating the contour, the nearest points clearly dominate. Again, the observer assigns the same points the same weights even when the entire stimulus is scaled by a factor of two. The locality of interpolation is not a constraint imposed by limitations in extra-foveal resolution.

Given any model of visual interpolation, its influence field is readily computed and compared to the empirically-observed influence fields of Exp. I. Since most "splining" or interpolation algorithm will adequately interpolate simple contours such as parabolas, it is difficult to test such models by comparison of predicted interpolation to observed interpolation by human observers.[3] The influence field provides a potentially powerful tool for testing hypotheses concerning human visual interpolation.

In Exp. III, we used the influence measure to characterize the membership of a displaced point in a sampled contour as a function of displacement. Our results indicate that we could readily measure the degree of membership of the isolated point in the contour in this way. We considered two possible interpretations of our result: the Graded Membership Hypothesis and the All-or-None Hypothesis. An analysis of observers' variability in setting provided some indication that membership of points in sampled contours is all-or-none (i.e. points do not have partial membership in a contour). These hypotheses needs to be tested by experiments specifically designed to test them.

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REFERENCES

1. G. Kanizsa, "Marzini quasi-percettivi in campi con stimolozione omogenea", *Rivista di Psicologia*, **49**, pp. 7-30, 1955
2. K. Koh, and L. T. Maloney, "Visual interpolation and extraction of linear and quadratic contours through discrete points", *Investigative Opthalmology and Vision Research*, Abstract, **29**, p. 408, 1988.
3. K. Koh, L. T. Maloney, "Test of regularization models: Interpolation and extrapolation of irregularly sampled contours in human vision", *Investigative Opthalmology and Vision Research*, Abstract, **30**, p. 458, 1989.
4. A. Blake, A. Zisserman, *Visual Reconstruction*, The MIT Press, Cambridge, 1987.
5. M. S. Landy, L. T. Maloney, E. B. Johnston, M. Young, "Measurement and modeling of depth cue combination: in defense of weak fusion", *Vision Research*, **35**, pp. 389-412, 1995.
6. W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, *Numerical recipes in C, 2nd edition*, Cambridge University Press, 1992.
7. F. R. Hampel, E. M. Ronchetti, P. J. Rousseeuw, A. S. Werner, *Robust Statistics, The Approach Based on Influence Functions*, John Wiley & Sons, Inc., New York, 1986.