

# Surface color perception as an inverse problem in biological vision

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## ABSTRACT

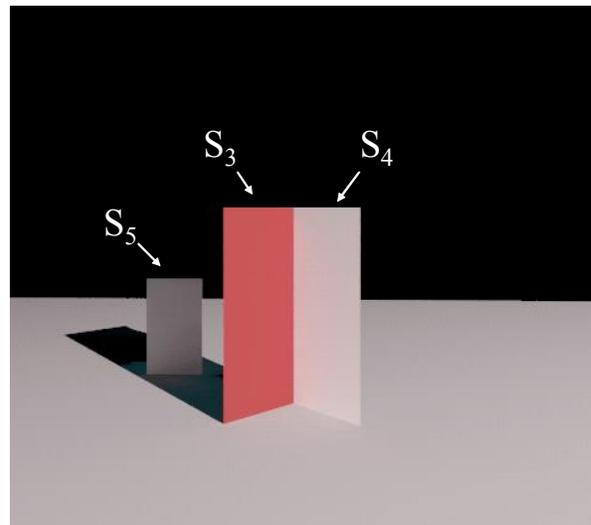
The spectral power distribution (SPD) of the light reflected from a matte surface patch in a three-dimensional complex scene depends not only on the surface reflectance of the patch but also on the SPD of the light incident on the patch. When there are multiple light sources in the scene that differ in location, SPD, and spatial extent, the SPD of the incident light depends on the location and the orientation of the patch. Recently, we have examined how well observers can recover surface color in rendered, binocularly-viewed scenes with more than one light source. To recover intrinsic surface color, observers must solve an *inverse problem*, effectively estimating the light sources present in the scene and the SPD of the light from each that reaches the surface patch. We will formulate the forward and inverse problems for surface color perception in three-dimensional scenes and present experimental evidence that human observers can solve such problems. We will also discuss how human observers estimate the spatial distribution of light sources and their chromaticities from the scene itself and how they might represent it.

**Keywords:** surface color perception, surface lightness perception, inter-reflection, spherical harmonics

## SURFACE COLOR PERCEPTION IN 3D SCENES

In an everyday scene, the light that reaches the eye from a matte surface patch depends on several factors. These include the surface properties of the patch itself, the location and orientation of the patch, the location and orientation of other surfaces that might serve as effective illuminants for the patch of interest, and the spatial distribution of light sources in the scene and their spectral properties. We will refer to this last as the *lighting model* of the scene. This list, long as it is, is still incomplete, but it gives us a starting point for the study of surface color perception in complex, three-dimensional scenes (Maloney, 1999). The simple virtual scene shown in Figure 1 illustrates some of these dependencies. The scene is illuminated by a combination of a punctate light source ( $E_1$ ) and a diffuse light source ( $E_2$ ), both achromatic. These together form the lighting model. The punctate light source is simulated to be behind the observer, on his right and is not directly visible. The scene comprises four matte surfaces, a ground plane and the three surfaces labeled  $S_3$ ,  $S_4$ , and  $S_5$ .

We are specifically interested in how biological visual systems extract information about surfaces (albedo, color) in such scenes. We are also interested in computational vision algorithms that model their performance. A biological visual system records the intensity and chromaticity of the light arriving from each point in the scene (see Maloney, 1999) but this information depends on



**Figure 1: A very simple scene composed of matte surfaces illuminated by a punctate light source and a diffuse light source.**

both the light incident on each surface as well as the properties of the surface. For example, the intensity of light arriving from  $S_5$  is less than that arriving from  $S_4$ , but much of that difference is due to differences in how the surfaces are illuminated:  $S_5$  is obviously in shadow with respect to the punctate light. But do the two surfaces have the same albedo? Answering this question correctly presupposes knowledge of the relative intensity of the light from the diffuse and the light from the punctate source absorbed and re-emitted by both surfaces. Similarly,  $S_4$  has a faint orange hue, the result of light emitted from  $S_3$  nearby.  $S_4$  is actually achromatic. Even in this very simple scene, working out the effective illumination incident on each surface is not easy while determining what this illumination is seems to be a necessary step in forming accurate estimates of surface properties.

**The Mondrian singularity.** What evidence there is suggests that biological visual systems embody solutions to problems of surface property estimation that we cannot yet duplicate algorithmically (Hurlbert, 1998; Maloney, 1999; Gilchrist et al., 1999; Jacobs, 1981, 1990, 1993; Lythgoe, 1979). Almost all previous experimental research in surface color perception concerns scenes made up of surface patches confined to a plane perpendicular to the line of sight, effectively illuminated by a diffuse light source ('flat world' in the terminology of Maloney, 1999). These scenes are often referred to as Mondrians (after Land & McCann, 1971). Maloney (1999) notes that there seem to be very few cues in such scenes that would permit a visual system, biological or computational, to separate the spectral properties of the illuminant from the spectral properties of the surfaces in the scene. Many computational algorithms that estimate surface properties corresponding to color make use of cues that are not available in 'flat world' but that are common in everyday scenes (to give two examples, specular highlights and shadows). Maloney & Yang (2001; Yang & Shevell, 2002, 2003) have demonstrated that one of these candidate cues (specular highlights) affects surface color perception.

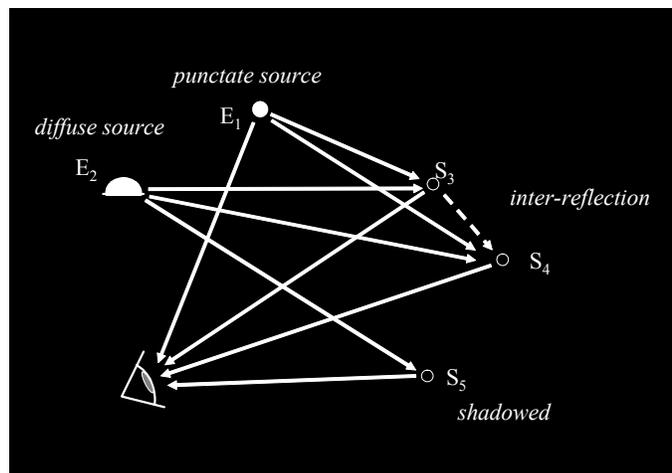
The underlying theme of Maloney (1999) is that researchers have expended considerable effort to study biological surface color perception under circumstances where it does not and perhaps cannot function very well, a paradox we refer to as the *Mondrian singularity*. An examination of surface color perception in more complex scenes might give better insight into the range of estimation problems that biological color vision can solve. In this proceedings paper, we examine human color vision in a range of three-dimensional scenes with spatially and spectrally non-uniform lighting that we refer to as *matte world*. Our results and results from other laboratories (notably Gilchrist, 1977; Gilchrist, 1980; Gilchrist et al, 1999; Brainard, Brunt & Speigle, 1997; Brainard, 1998) suggest that human color vision is well-equipped to solve these apparently more complex problems in surface color perception. We will first present some of these results and then discuss how biological vision systems may achieve the performance they achieve by solving a series of inverse problems.

## THE FORWARD AND INVERSE PROBLEMS OF SURFACE COLOR PERCEPTION

**Matte world.** In Fig. 1, we illustrated these factors in an extremely simple scene containing three matte surfaces in addition to a matte ground plane. The three surfaces are assumed to be small enough in visual extent to appear homogeneous to the observer. Surfaces  $S_3$  and  $S_4$  are illuminated by both the punctate and diffuse components of the lighting model while surface  $S_5$  is illuminated only by the diffuse component. Surface  $S_4$  is also illuminated by light that has been absorbed and re-emitted by surface  $S_3$ . We should also take into account light absorbed and emitted from the ground plane and light resulting from higher-order reflections between  $S_3$  and  $S_4$ . However, let us stop with these three surfaces, limited inter-reflection, and the specified

lighting model and represent the flow of light among them as a *light-flow graph* (Fig. 2).

A light-flow graph is a convenient representation of how light flows within a scene consisting of only matte surfaces. The nodes in a



**Figure 2: The light-flow diagram corresponding to a subset of the surfaces in Figure 1.**

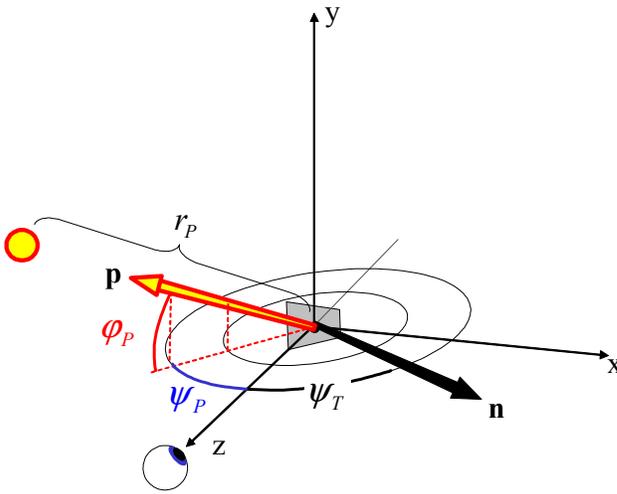
light-flow graph represent light sources (punctate sources

represented by a solid circle, diffuse by a solid semi-circle) or matte surface patches (hollow circles). As just noted, the surface patches are assumed to be sufficiently small that the light emitted in all directions does not depend on location within the patch.

**The forward problem.** The *forward problem* for a light-flow graph is to predict the light that will arrive at an eye or camera placed at a specified location and orientation in the scene. This problem is the problem solved by standard rendering and ray tracing methods employed in computer graphics (see Larson & Shakespeare, 1996). The rendering package must determine the intensity and spectral composition of the light that passes along each of the arrows in Fig. 2, taking into the locations and specifications of light sources, surface orientation, surface albedo, and more. Typical rendering packages allow for non-matte surfaces, transparency, and active media such as mist (Larson

& Shakespeare, 1996). However, the forward problem posed by a small number of matte surfaces in a scene with a non-diffuse lighting model is sufficient for our purposes. What we are interested in here is a series of inverse problems implicit in Figures 1 and 2 that we address below. Estimating surface properties in ‘matte world’ is equivalent to solving these inverse problems.

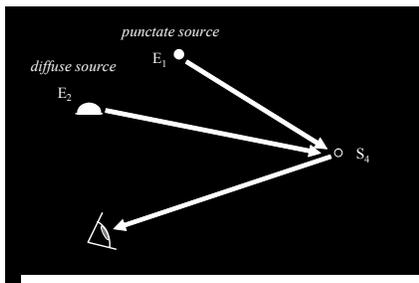
**Spatial coordinate system.** We used a spherical coordinate system  $(\psi, \phi, r)$  based on a Cartesian coordinate system to describe the geometry of the location in any scene (Fig. 3). In the Cartesian system  $(x, y, z)$ , the z-axis will fall along the observer’s line of sight, the y-axis is vertical, the x-axis horizontal as shown. In the spherical coordinate system, a point in the three dimensional space is denoted by three numbers  $(\psi, \phi, r)$ :  $r$  is the distance of the point from the origin,  $\psi$  is the angle



**Figure 3: Spatial coordinate system.**

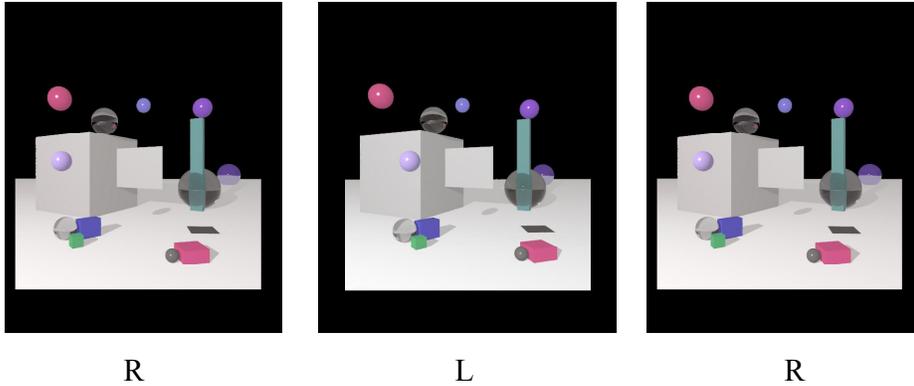
between the observer’s line of sight (z-axis) and the projection of the point on the horizontal plane (xz plane),  $\phi$  is the angle between the horizontal plane and the line connecting the origin and the point. We refer to  $\psi$  as *azimuth*,  $\phi$  as *elevation*. In scenes containing a punctate light, we will denote the position of the light by  $(\psi_p, \phi_p, r_p)$  and the direction to the light by  $(\psi_p, \phi_p)$ . We will denote the orientation of a test patch (the surface whose color or albedo the observer judges) by the azimuth and elevation of the normal  $n$  to the surface:  $(\psi_T, \phi_T)$ .

### INVERSE PROBLEM 1: ALBEDO ESTIMATION



**Figure 4: Inverse problem 1.**

Boyaci et al (2003) examined asymmetric lightness matching in rendered scenes illuminated by a combination of achromatic diffuse and punctate light sources. The forward problem is captured by the light flow diagram in Figure 4. There are two achromatic light sources, punctate and diffuse, and an achromatic test patch, S4. The observer’s task is to estimate the albedo of the achromatic matte test patch, the proportion of light that it reflects. The amount of light from the punctate source depends on the orientation of the test patch with respect to the punctate source (see below) and the observer can estimate the albedo of the test patch only by discounting this effect of orientation. The forward problem is



**Figure 5: The left and right images of a stereo pair in Experiment 1. The test patch is at the center of the image. The left and right images are marked L and R and, for proper fusion, should be viewed by the left and right eye, respectively**

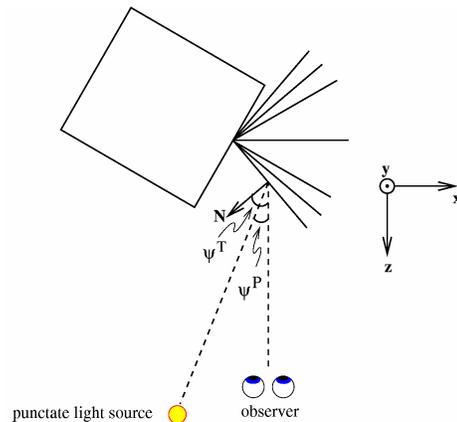
straightforward and could be solved with a calculator. The inverse problem, however, is what the observer's visual system must solve.

All viewing was binocular and Fig. 5 contains a typical stereo image pair. The test patch is the small achromatic patch in the center of the scene. All images were rendered using a standard rendering package RADIANCE (Larson & Shakespeare, 1996) and stereo image pairs were created by rendering the same scene twice from different viewpoints corresponding to the locations of the observer's eyes. The punctate light source was out of sight, behind the observer and he was given no information about its location other than what could be deduced from the scenes presented. In pilot testing we found that, if the scene contained only the test surface, the observer acted as if the position of the punctate light source was unknown. The large number of extra objects present in the scene are potential cues to the spatial and spectral distribution of the illumination (the lighting model). Some of these objects are specular, and could potentially inform the visual system about the distribution of light sources throughout the scene. We return to this issue in a later section.

The luminance of the punctate source is denoted by  $E_p$  and the spectral power distribution of the diffuse light by  $E_D$ . The punctate source is placed far enough from the test patch so that we can regard it as being at infinity. The direction to the punctate light source is, in the notation described above,  $(\psi_p, \phi_p)$ .

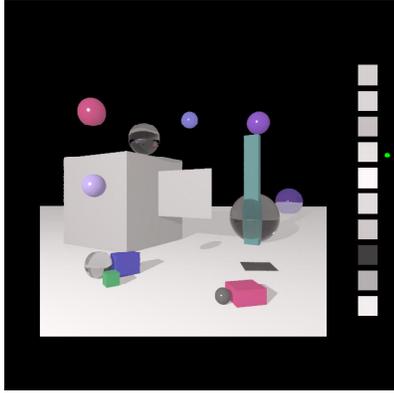
From trial to trial, the experimenters randomly varied the orientation  $(\psi_T, \phi_T)$  of the test patch to one of seven values shown in Figure 6. Only  $\psi_T$  was varied with  $\phi_T = 0^\circ$  always.

Boyaci et al assumed that matte surfaces in the scene could be modeled as Lambertian surfaces. In the Lambertian model, the intensity of emitted light does not depend on the direction to the viewer, so long as the viewer and the light source are on the same side of the surface. It does depend upon  $\theta$ , the angle between the direction to the punctate source and the surface normal (the angle of incidence).



**Figure 6: Possible orientations of the test patch (view from above).**

**The observer's task.** The observer's task was to select a reference surface from a set of reference surfaces (Fig. 7) that matched the apparent albedo of the test surface. The observer clicked on the left and right button of a mouse to move a small green dot up and down the scale. When the green dot was next to the observer's choice, the observer clicked on the middle button to record his choice. The reference surfaces spanned the gamut from black to white and the order of the surfaces in the scale was randomized anew on each trial.



**Figure 7: The observer's task was to select the reference surface on the scale to the right that matched the test patch in albedo.**

How will the observer's settings change as function of the orientation of the test patch? One possibility is that they will not change. This observer effectively ignores orientation in arriving at estimates of albedo. A second possibility is that the observer will correctly select the reference surface that does match in albedo. To do so, the observer must effectively determine how the illumination of the test patch changes with orientation. To see what that entails, we must work out the forward problem for the conditions of the experiment and then examine what is entailed in solving the inverse problem.

**The forward problem.** Boyaci et al (2003) derived a model of the luminance,  $L$ , that reaches the observer's eye as a function of the surface albedo,  $\alpha$ , and a geometric factor,  $\Gamma(\theta, \pi)$ , explained below,

$$L = \frac{E}{\Gamma(\theta, \pi)} \alpha \quad (1)$$

The total luminance,  $E = E_p + E_D$  is the maximum possible luminance that could reach the eye from punctate and diffuse sources reflected from a chip of albedo  $\alpha = 1$ . The parameter  $\pi = E_p/E$  is the punctate-total ratio. It captures the relative intensities of the punctate and diffuse components of the light. When  $\pi$  is 0, for example, the light is perfectly diffuse, and when it is 1, the light is punctate only. The geometric factor, defined as,

$$\begin{aligned} \Gamma(\theta, \pi) &= (\pi \cos \theta + 1 - \pi)^{-1}, & 0^\circ \leq \theta < 90^\circ \\ &= (1 - \pi)^{-1}, & \theta \geq 90^\circ. \end{aligned} \quad (2)$$

characterizes how the location of the punctate source and the test surface orientation affect the intensity of the light reaching the eye. The angle  $\theta$  is, as defined above, the angle of incidence of the light from the punctate source. The cosine term is a consequence of Lambert's Law. When  $\theta$  is greater than  $90^\circ$  the test patch is in shadow with respect to the punctate source. When  $\theta$  is  $0^\circ$ , the test patch directly faces the punctate source and  $\Gamma(\theta, \pi) = 1$ . Eq. 2 above captures the forward problem. We can use it to compute the light arriving at the eye given a specification of the lighting model and the albedo and orientation of the test patch.

**The inverse problem.** We can rearrange Eq. 1 to specify the inverse problem:

$$\alpha = \frac{L}{E} \Gamma(\theta, \pi) \quad (3)$$

Given knowledge of the parameters  $\theta$ ,  $\pi$ , and  $E$ , we can use Eq. (3) to estimate surface albedo given the luminance of light reaching the retinas from the test patch,  $L$ .

**Equivalent lighting models.** In order to use Eq. (3) to estimate surface albedo accurately, a visual system needs accurate estimates of all of the quantities on the right-hand side of the equation. The intensity of light arriving

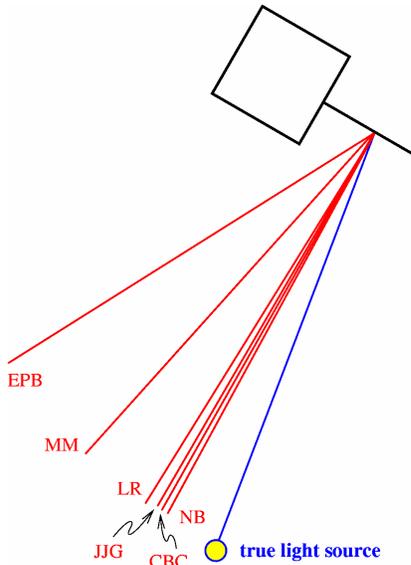
from the surface  $L$  is what is measured by the sensors of the visual system. Estimates of the orientation  $(\psi_T, \phi_T)$  of the surface in a scene are needed to work out estimates of the angle  $\theta$  between the surface normal and the punctate light source. The experiment of Boyaci et al also included a control task that allowed them to verify that each observer's perception of surface orientation was close to the correct values. The remaining values needed are all part of the lighting model: the direction to the punctate light source  $(\psi_P, \phi_P)$ , the total light intensity  $E$  and the punctate-total intensity ratio  $\pi$ . Part of the task of the visual system is to work out estimates of these parameters (or the parameters of an equivalent parameterization). Given estimates of the lighting model parameters  $[\hat{E}, \hat{\psi}_P, \hat{\phi}_P, \hat{\pi}]$ , together with estimates of test surface orientation and the intensity of light emitted by the surface,

$$\hat{\alpha} = \frac{\hat{L}}{\hat{E}} \Gamma(\hat{\theta}, \hat{\pi}) \quad (4)$$

where it is easy to show that

$$\hat{\theta} = \cos^{-1}(\cos \hat{\phi}_T \cos \hat{\phi}_P \cos \hat{\psi}_T \cos \hat{\psi}_P + \sin \hat{\phi}_T \sin \hat{\phi}_P + \cos \hat{\phi}_T \cos \hat{\phi}_P \sin \hat{\psi}_T \sin \hat{\psi}_P) \quad (5)$$

We refer to the vector of parameter estimates  $[\hat{E}, \hat{\psi}_P, \hat{\phi}_P, \hat{\pi}]$  as the visual system's *equivalent lighting model*, an estimate of the true lighting model present in the scene.



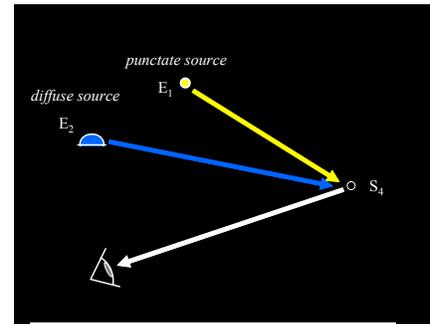
**Figure 8: Estimates of the azimuth of the punctate light source.**

**Results.** The results of Boyaci et al indicated that observers partially compensated for test patch orientation in the scenes employed and that their settings were remarkably consistent with the Equivalent Lighting Model (ELM) just described. By fitting the ELM to each observer's data, Boyaci et al obtained an estimate of each of the observers' ELM parameters. The estimates of  $\hat{\psi}_P$  for six observers are shown in Figure 8 together with the true value,  $\psi_P$ . It is clear that we can recover a crude estimate of light direction based only on observers' matching data in the experiment, demonstrating that their matching performance takes into account the spatial distribution of illumination in the scene. These results were replicated by Ripamonti et al (2004) in a more complex design using physical surfaces and lights (not rendered scenes).

Two points: First, we do not claim that the observer's visual system uses precisely the parameterization of the equivalent lighting model that we developed above  $([\hat{E}, \hat{\psi}_P, \hat{\phi}_P, \hat{\pi}])$ . We claim only that, the observer's visual system represents the spatial distribution of light in the scene and that this representation contains information equivalent to  $[\hat{E}, \hat{\psi}_P, \hat{\phi}_P, \hat{\pi}]$ . Second, a related question is, how does the visual system form estimates of the parameters in the equivalent light model?

## INVERSE PROBLEM 2 : CHROMATICITY ESTIMATION

**Color.** The second inverse problem we consider is estimation of surface color in scenes with multiple light sources differing in chromaticity. The light-flow diagram of the forward problem is shown in Figure 9. The lighting model consists of a punctate yellow light source (‘sun’) and a diffuse blue light source (‘sky’) and the test surface is illuminated by a mixture of the two that depends on the orientation of the test surface as well as the lighting model. The observer’s task is to set the test patch to be neutral (grey).



**Figure 9: The second inverse problem.**

This inverse problem is dual to the first in the sense that, in the first, the observer’s visual system had to compensate for the contribution of diffuse and punctate light sources that were achromatic in estimating the albedo of a test patch that was neutral in color appearance. Now the observer must work out the blue-yellow balance of light incident on a test patch implicit in the spatial organization of the scene and set the chromaticity of the light emitted by the surface to be consistent with that of an achromatic surface. The lighting model in this second case is more complex than that we developed in considering the first inverse problem. The observer’s visual system must now estimate not only the location of the punctate sources, the intensities of the two sources, but also their chromaticities.

The experimental design was similar to that of Boyaci et al (2003) except that the test patch was varied in both azimuth and elevation ( $\psi_T, \phi_T$ ) from trial to trial and the direction to the punctate source ( $\psi_P, \phi_P$ ) was also varied: it could be behind the observer on his left or on his right. A typical scene is shown in Figure 10. Note how the chromaticity varies in going from fully-illuminated surfaces to shadow. The test patch ‘floats’ in front of the other objects in the scene. Again, these objects are potential cues that the visual system may use to estimate the lighting model.



**Figure 10: An example of a stereo image pair from Boyaci et al (2004). The format is identical to that of Figure 5.**

**Results.** Boyaci et al (2004) fit an equivalent lighting model to observers’ achromatic settings. The estimates of the azimuth  $\psi_P$  and the elevation  $\phi_P$  of the punctate yellow light source are plotted in Figure 11 along with the true values (recall that two values of azimuth were employed in the experiment). Boyaci et al were able to recover crude estimates of light source direction from the observer’s surface color estimates in scenes where the spectral distribution of the light was not spatially uniform. These are plotted in Figure 11. There are four estimates of azimuth (one for each observer) for the light source on the left and four for the light source on the right. There are eight corresponding estimates of elevation. With one exception the eight estimates of elevation are within 10 degrees of the true values. The estimates of elevation are tightly clustered, within 10 degrees of the true values, with one exception.

The outcome of the experiment in Boyaci et al (2004) together with the results of Boyaci et al (2003) imply that the observer's visual system effectively develops an equivalent lighting model of the spatial and spectral distribution of illumination within a scene and uses this model in estimating albedo and surface color.

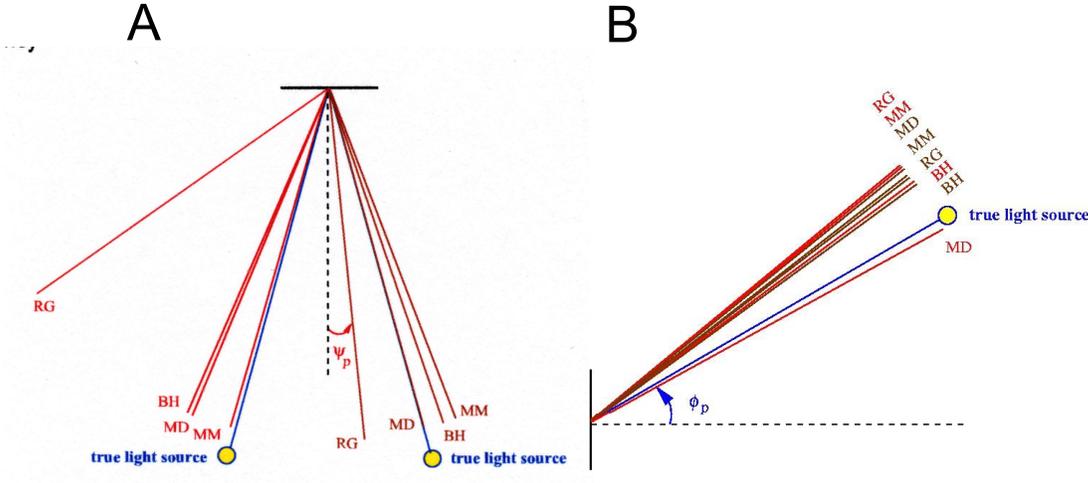


Figure 11: A. Fitted estimates of the azimuth of the punctate light for four observers and the corresponding true azimuths when the punctate light was on the observer's left or right. B. Estimates of the elevation of the punctate light for the same observers.

INVERSE PROBLEM 3: DISCOUNTING INTER-REFLECTION

*Inter-reflection.* So far we have considered lighting models where all of the light sources are effectively placed at infinity. In complex scenes, the light emitted by one surface can fall on a second, becoming, in effect, a component of the illumination incident on the second. The corresponding light-flow diagram is shown in Figure 12.

In Figure 13A, for example, the light gray matte *test surface* marked *T* absorbs light that reaches it directly from the single light source in the scene. It also absorbs light that arrives from the same light source but only after being absorbed and re-emitted from the nearby orange surface marked *C* (Fig. 13B). Part of the light absorbed and re-emitted by the test surface will in turn be absorbed and re-radiated by the orange surface, initiating an infinite series of interreflections between the surfaces. If we denote the spectral power distribution of the original illuminant by  $E_{(0)}(\lambda)$  and the surface reflectance

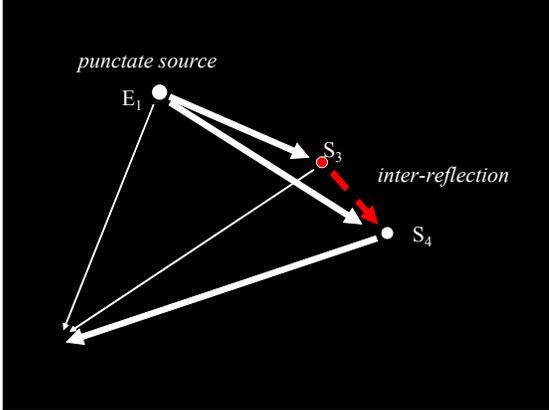


Figure 12: The third inverse problem.

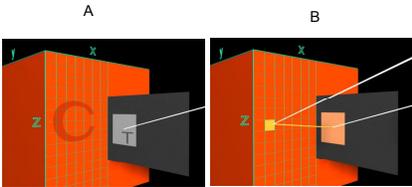
functions of the two surfaces by  $S_C(\lambda)$  (cube) and  $S_T(\lambda)$  (test surface), then the light emitted from any specified small region of the surface toward the observer can be written in the form  $E(\lambda)S_T(\lambda)$  where

$$E(\lambda) = \sum_{i=0}^{\infty} \gamma_i E_{(i)}(\lambda) \tag{6}$$

is the *effective illuminant*. It is the weighted sum of the direct illumination,  $E_{(0)}(\lambda)$ , and the inter-reflected illuminants,

$$E_{(1)}(\lambda) = E_{(0)}(\lambda)S_C(\lambda);$$

$$E_{(i+2)}(\lambda) = E_{(i)}(\lambda)S_T(\lambda)S_C(\lambda) \quad i = 0, 1, \dots \tag{7}$$



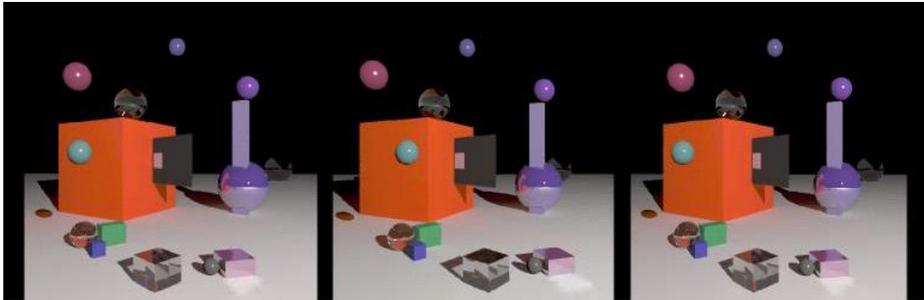
**Figure 13: inter-reflection.**

The *geometric factors*  $\gamma_i$  are determined by the sizes and shapes of the two surfaces, their separation, and their orientations with respect to one another and with respect to the primary light source. We will assume that they do not depend on wavelength  $\lambda$  in the electromagnetic spectrum. To make stable estimates of the surface color and albedo of a surface in a scene, independent of scene layout or illumination, a visual system must discount the effective illuminant  $E(\lambda)$  at each point in the scene, even when that illumination

includes contributions from other surfaces in the scene. There is some evidence that human observers do so, if only partly and imperfectly (Bloj, Kersten & Hurlbert, 1999). Doerschner, Boyaci & Maloney (2004) examined how observers estimate surface color in scenes similar to the one shown in Figure 14. The scene was illuminated by a single neutral punctate light source placed behind the observer.

**The observer’s task.** The observer was asked to set the color of the small test patch adjacent to the cube to appear to be a neutral grey (achromatic setting). The test patch is embedded in a dark grey rectangular surface to make it easier to judge the test patch orientation. The experimenter varied the angle between the test surface (including the dark grey rectangle in which it is embedded) and the large orange cube from trial to trial. The observer’s achromatic settings were fit to a equivalent illumination model of that took into account the inter-reflection between surfaces.

**Results.** Doerschner et al found that observers systematically but only partially discounted the effect of light reflected from the large orange cube in making achromatic settings. In effect, they consistently underestimated the contribution of inter-reflected light to the perceived chromaticity of the test patch, possibly by underestimating the area of the cube that contributed light to the test surface. However, the pattern of discounting as a function of the angle of the test patch was almost perfectly captured by the equivalent lighting model once this underestimation was taken into account.

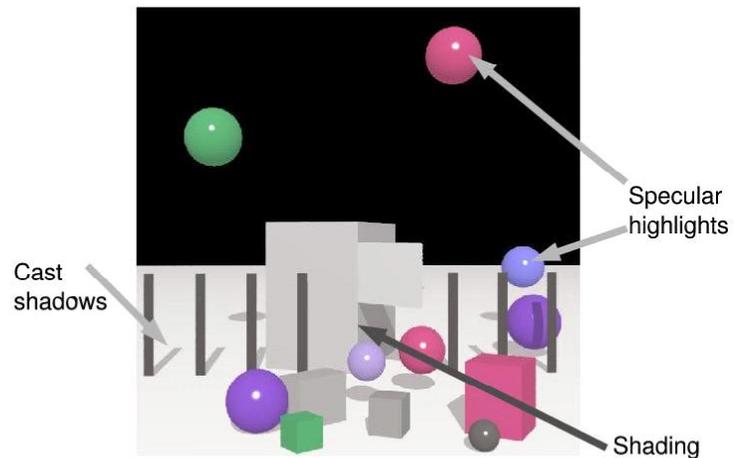


**Figure 14: A pair of stereo images from Doerschner et al (2004). Format as in Figure 5.**

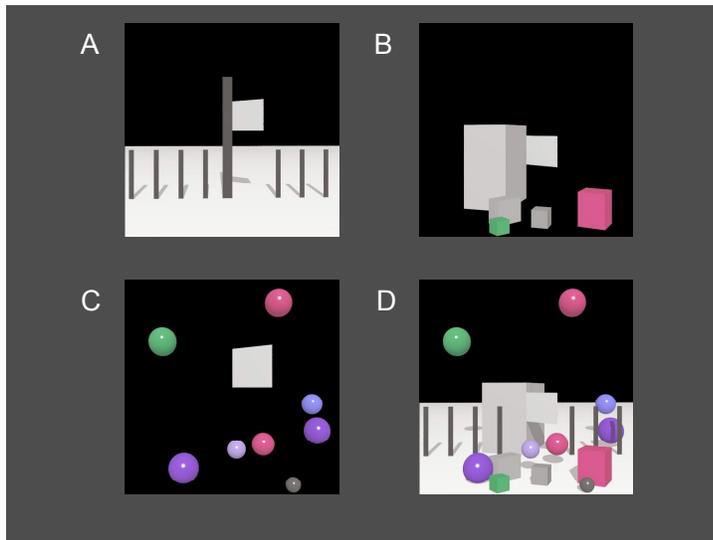
## CUES TO THE LIGHTING MODEL

The experimental results just summarized indicate that observers take into account the orientation of surfaces and the spatial layout of light sources in a scene and that they substantially compensate for both in estimating surface albedo and color, effectively solving a series of inverse problems. Our results motivate the following question concerning the estimation of the parameters of an equivalent lighting model. What cues (sources of information) within the scene does the visual system employ in estimating the spatial and spectral distribution of the light?

The scenes we used contained several candidate cues to the spatial and spectral distribution of illumination: specular highlights, cast shadows, and shading, as illustrated in Figure 15. In previous work, Yang & Maloney (2001; Yang & Shevell, 2002, 2003) demonstrated that the visual system makes use of specular highlights in estimating the chromaticity of the illuminant. Boyaci et al (in preparation) report an experiment that tested directly whether observers could use each of these cues in estimating the spatial distribution of light in a scene where the illuminant consisted of a neutral punctate source and a neutral diffuse source (as in Boyaci et al, 2003).



**Figure 15: Candidate cues to the illuminant.**



**Figure 16: Candidate cues (right images of stereo pairs). (A) cast shadows only, (B) shading only, (C) specular highlights only, (D) all three cues combined.**

Observers saw four different kinds of scenes (Figure 16). The scenes were binocularly presented as in the previous experiments described. The first three kinds contained only one cue to the spatial distribution of the illuminant (Figure 16A-C). The last kind contained all three types of cues, effectively superimposed as in Figure 3D. The observers task was as in Boyaci et al (2003): match the albedo of a test surface to one of the reference surfaces in a scale.

**Results.** We fit equivalent lighting models to observers' data. The key analysis was to test whether observers' estimates of the punctate-diffuse ratio  $\tau$  were greater than 0. This would imply that they had detected the punctate source and estimated its direction within the scene. We concluded that observers did use the available cue in all scenes containing one kind of cue and that some observers used more than one cue when more than one cue was available (Figure 16D).

## CONCLUSION

We have presented a series of experimental tests of human color vision in complex, three-dimensional scenes. Each test was formulated as an inverse problem (where the forward problem was effectively simulation of light flow within the scene). Solving the inverse problem required that the observer's visual system effectively estimate the lighting model for the scenes employed in the experiment.

We found that observers made judgments of surface color and albedo that indicated that they had partially but systematically solved the corresponding inverse problem. The deviations from veridical performance were consistent with the hypothesis that observers had misestimated the location, chromaticity, and intensity of light sources within the scene and had used these misestimates in solving the inverse problem.

Much previous work on color vision has employed scenes consisting of a uniformly-illuminated flat surface covered with matte surface patches all perpendicular to the observer's line of sight, a Mondrian. The inverse problem for Mondrian scenes is very simple: its solution only presupposes an estimate of the chromaticity and intensity of the illuminant. It is therefore somewhat surprising that, given only the retinal images corresponding to a Mondrian scene, it is not possible to estimate the chromaticity of the illuminant in the scene without additional constraints and impossible to solve the corresponding inverse problem (Maloney, 1999). The inverse problem is ill-posed, a result we refer to as the 'Mondrian singularity'.

The scenes in the experiments reported here correspond to more complex inverse problems, where accurate estimation of the color and albedo of surfaces within the scene presupposes that the visual system effectively estimates more about the spatial and spectral distributions of the illuminant. However, these scenes also contained additional candidate cues that specifically provide information about the lighting model. These inverse problems are not ill-posed and we find that human observers seem able to use the illuminant cues that we provide to solve them.

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