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37

Abstract

38 We discuss behavioral studies directed at understanding how probability
39 information is represented in motor and economic tasks. By formulating the behavioral
40 tasks in the language of statistical decision theory, we can compare performance in
41 equivalent tasks in different domains. Subjects in traditional economic decision-making
42 tasks often misrepresent the probability of rare events and typically fail to maximize
43 expected gain. In contrast, subjects in mathematically equivalent movement tasks often
44 choose movement strategies that come close to maximizing expected gain. We discuss
45 the implications of these different outcomes, noting the evident differences between the
46 source of uncertainty and how information about uncertainty is acquired in motor and
47 economic tasks.

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49 ***Risky decisions and movement planning***

50 Uncertainty plays a fundamental role in perception, cognition and motor control
51 and a wide variety of biological tasks can be formulated in statistical terms. How the
52 organism combines sensory information from many different sources (“cues”) is currently
53 an active area of research and several groups have proposed [1, 2] that perceptual
54 estimation of properties of the environment can be framed within Bayesian decision
55 theory, a special case of statistical decision theory [3]. We will show that framing
56 behavioral tasks in the language of statistical decision theory enables a comparison of
57 performance between motor tasks and decision making under risk.

58 Research concerning decision-making seeks to understand how subjects choose
59 between discrete plans of action that have economic consequences [4]. A subject might
60 be given a choice between a 10% chance of \$5000 (and otherwise nothing) and a 95%
61 percent chance of \$300 (and otherwise nothing). These possible choices can be written
62 in compact form as *lotteries* $L_1 = [0.1, \$5000; 0.9, \$0]$ and
63 $L_2 = [0.95, \$300; 0.05, \$0]$. If subjects are told probabilities, they are making
64 “decisions under risk” and otherwise, “decisions under uncertainty” [5]. Here, we are
65 concerned primarily with the former.

66 Of course, most subjects would prefer to receive \$5000 rather than \$300, or
67 \$300 rather than \$0. The key difficulty in making such decisions is that no plan of action
68 (lottery) available to the subject guarantees any specific outcome.

69 We review recent experimental work in movement planning [6-9] in which
70 humans perform speeded movements towards displays with regions which, if touched,
71 lead to monetary rewards and penalties (Box 1). Our work shows that humans do very
72 well in making these complex decisions in motor form. This outcome is particularly

73 surprising since humans typically do not do well in equivalent economic decision-making
74 tasks as we describe next.

75 ***Sub-optimal economic decisions***

76 Human performance in decision making under risk is markedly sub-optimal and
77 fraught with cognitive biases [4] that result in serious deficits in performance. Patterned
78 deviations from maximizing expected gain include a tendency to frame outcomes in
79 terms of losses and gains with an exaggerated aversion to losses [10] and to exaggerate
80 the weight given to low-probability outcomes [11, 12]. This latter property parallels the
81 human tendency to overestimate the relative frequencies of rare events [13, 14]. This
82 exaggeration of the frequency of low-frequency events is observed in many but not all
83 decision-making studies [15]. These behaviors are typically modeled by Prospect Theory
84 by introducing a probability weighting function and by assuming that subjects maximize a
85 tradeoff between losses and gains [10, 12].

86 ***Motor tasks equivalent to decision making under risk***

87 Recent work in motor control [9] formulates movement planning in terms of
88 statistical decision theory, effectively converting the problem of movement planning to a
89 decision among lotteries that is mathematically equivalent to decision making under risk.
90 We can compare performance in economic decision-making tasks with performance in
91 equivalent motor tasks and also study how organisms represent value and uncertainty
92 and make decisions in very different domains [16-22].

93 In Figure 1a, we illustrate the task and show one of the target-penalty
94 configurations used in [6]. The rules of the task are very simple. The configuration
95 appears on a display screen a short distance in front of the subject. The subject must
96 reach out and hit somewhere on the display screen in a short period of time. The subject

97 knows that hits within the green circle result in a monetary payoff but hits within the red
98 result in a loss of money. The amounts vary with experimental condition but in the
99 example in Figure 1a they are 2.5 cents and 12.5 cents, respectively.

100 -----
101 Figure 1 about here
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103 At the speed the subject is forced to move, movement can't be completely
104 controlled: even if the subject aims at the center of the green circle there's a real chance
105 of missing it. And, if the subject aims too close to the center of the green circle, there is a
106 risk of hitting inside the red. Where should the subject aim?

107 In Box 1, we show how to interpret the subject's choice of aim point as a choice
108 among lotteries and how to determine the aim point that maximizes expected gain. An
109 economist would use the term "utility" where a researcher in motor control would opt for
110 "cost" or "biological cost" and a statistician would use "loss" or "loss function." Adopting a
111 field-neutral term, we refer to rewards and penalties associated with outcomes as
112 "gains", positive or negative.

113 We were surprised to discover that, in this decision task in motor form,
114 participants typically chose visuo-motor plans that came close to maximizing expected
115 gain. In Figure 1b we plot the subject's displacement in the horizontal direction away
116 from the center of the green circle versus the displacement that would maximize
117 expected gain, combining data across several experimental conditions varying the
118 penalty amount and distance between target and penalty circles [6, 7].

119 -----
120 Box 1 and its figure about here
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122 Additional research has extended this conclusion to tasks that involve precise
123 timing and tradeoff between movement time and reward [23, 24] and to tasks involving
124 rapid choices between possible movement targets [9]. Generally, human subjects

125 choose strategies that come close to maximizing expected gain in motor tasks with
126 changing stochastic variability [8, 25] or combining noisy sensory input with prior
127 information [26-29].

128 These results have implications for our understanding of movement planning and
129 motor control. Typical computational approaches to modeling movement planning take
130 the form of an optimization problem in which the cost function to be minimized is
131 biomechanical and the optimization goal is to minimize some measure of stress on the
132 muscles and joints. These models differ primarily in the choice of the cost function.
133 Possible biomechanical cost functions include measures of joint mobility [30, 31], muscle
134 tension changes [32], mean squared rate of change of acceleration [33], mean torque
135 change [34], total energy expenditure [35] and peak work [36]. These biomechanical
136 models have successfully been applied to modeling reaching movements as following a
137 nearly straight path with a bell-shaped velocity profile and also capture the human ability
138 to adapt to forces applied during movement execution [37]. We emphasize that these
139 models cannot be used to predict subjects' performance in our movement decision tasks
140 where performance also depends on externally imposed rewards and penalties.
141 Moreover, subjects came close to maximizing expected gain with arbitrary, novel
142 penalties and rewards imposed on outcomes by the experimenter.

143 Subjects do not always come close to maximizing expected gain in movement
144 planning, e.g. when the number of penalty/reward regions is increased [38] and when
145 the reward or penalty received is stochastic rather than determined by outcome of the
146 subject's movement [39]. Furthermore, when the penalty is so high that the aim point
147 maximizing expected gain lies *outside* of the target, results suggest that subjects prefer
148 not to aim outside of the target that they are trying to hit [8]. Thus, while there is a
149 collection of motor tasks, described above, where performance is remarkably good, we
150 cannot simply claim that performance in any task with a speeded motor response will

177 To see how unusual this outcome is, consider applying a simple reinforcement-
178 learning model according to which the aim point is adjusted gradually in response to
179 rewards and penalties incurred [40-42]. In the absence of any reward or penalty, a
180 learning model based on reward and penalty would predict that the subject should aim at
181 the center of the green circle, just as in the training trials. The subject would then
182 gradually alter the aim point in response to rewards and penalties incurred until the final
183 aim point maximized expected gain (Figure 2b).

184 However, examination of the initial trials of the decision phase of the experiment
185 suggests that subjects immediately changed their aim point from that used in training to
186 that required to trade off the probabilities of hitting the reward and penalty regions
187 (Figure 2a). This apparent lack of learning is of great interest in that it suggests that,
188 while subjects certainly learned to carry out the motor task in the training phases of
189 these experiments, and learned their own motor uncertainty, they seemed not to need
190 further experience with the decision-making task to perform as well as they did, applying
191 the knowledge of that motor uncertainty to new situations. The trends in performance
192 found with repetition of economic-decision tasks seem absent in equivalent movement-
193 planning tasks.

194 The contrast between success in “movement planning under risk” and decision
195 making under risk is heightened by the realization that, in decision making under risk,
196 subjects are *told* the exact probabilities of outcomes and thus have perfect knowledge of
197 how their choice changes the probability of attaining each outcome. The knowledge of
198 probabilities in equivalent motor tasks is never communicated explicitly and thus could
199 equal but never exceed the knowledge available under decision making under risk. Yet
200 the lack of learning in these motor tasks suggests that humans are able to estimate the
201 probabilities of each outcome associated with any given aim point due to motor
202 uncertainty and make use of this knowledge to improve their performance [8]. There is

203 mounting evidence that decision makers behave differently if knowledge of probabilities
204 is gained through “experience” (see Box 2). Our results add a new dimension to what
205 kinds of experience lead to enhanced decision making.

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207 Box 2 and its figure about here
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209 There is growing interest in analyzing brain activity in response to manipulations
210 of various components of decision-making under risk or uncertainty in human subjects
211 (for more extensive reviews, see [16, 17, 19, 20]). The work described here effectively
212 opens a second window on neural processing of uncertainty and value by allowing us to
213 present exactly the same decision problems in different guises.

214 ***Statistical decision theory: future directions***

215 The motor tasks we have considered are very simple, a reaching movement to
216 touch a target. Even these very simple motor tasks correspond to complicated choices
217 among lotteries. We close by illustrating that the underlying statistical framework,
218 statistical decision theory, can be used to model complex movement tasks shaped by
219 externally imposed rewards and penalties and where visual uncertainty can play a larger
220 part (Box 3).

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222 Box 3 and its figures about here
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224 Using the methods described there, we can translate visuo-motor and economic
225 decision making tasks into a common mathematical language. We can frame movement
226 in economic terms or translate economic tasks into equivalent visuo-motor tasks. Given
227 the societal consequences associated with failures of decision making in economic,
228 military, medical and legal contexts, it is worth investigating decision tasks in domains
229 where we seem to do very well.

230

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Box 1: Constructing Motor Lotteries

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In the main text, we described a movement task equivalent to decision making under risk. On each trial, subjects reach out and touch a computer screen within a short period of time (e.g. 700 ms). Hits inside a green target region displayed on a computer screen yield a gain of 2.5 cents; accidental hits inside a nearby red penalty region incur losses of 12.5 cents. Movements that do not reach the screen within the time limit are heavily penalized (following training they almost never occur). The subject is not completely under control of the movement outcome [43]. The black dots in the figures mark simulated outcomes of attempts to hit the diamond marking the target center (Figure 1a) or the diamond closer to the target edge (Figure 1b).

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Box 1's Figure 1 about here

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A movement that reaches the screen within the time limit can end in one of four possible regions: penalty only (Region R_1 , gain $G_1 = -12.5$), target/penalty overlap (Region R_2 , gain $G_2 = -10$), target only (Region R_3 , gain $G_3 = 2.5$), or neither/background (Region R_4 , gain $G_4 = 0$). In evaluating movement plans in this task, visuo-motor plans that lead to a touch on the screen within the time limit differ only to the extent that they affect the probability $P_s(R_i)$ of hitting each of the four regions $R_i, i = 1, \dots, 4$. The combination of event probabilities $P_s(R_i)$ resulting from a particular visuo-motor plan (aim point) S and associated gains G_i form a lottery,

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$$L(s) = [P_s(R_1), G_1; P_s(R_2), G_2; P_s(R_3), G_3; P_s(R_4), G_4]. \quad (1)$$

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An alternative visuo-motor plan S' corresponds to a second lottery,

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$$L(s') = [P_{s'}(R_1), G_1; P_{s'}(R_2), G_2; P_{s'}(R_3), G_3; P_{s'}(R_4), G_4]. \quad (2)$$

259 Each lottery corresponds to an aim point. The lottery corresponding to the aim point in
260 Figure 1a has an expected gain of -2.8 cents per trial (the subject loses money on
261 average), the expected gain associated with the aim point in Figure 1b is .78 cents per
262 trial. Obviously, the aim point in Figure 1b offers higher expected gain. In planning
263 movement in this task, subjects effectively choose among not just these two aim points
264 but infinitely many aim points (lotteries). They are engaged in a continuous decision-
265 making task of extraordinary complexity, and it is a task that is performed every time
266 they move.

267 In Figure 1c we plot the expected gain associated with every possible aim point
268 and highlight four of them. The aim point corresponding to the yellow diamond
269 maximizes expected gain for this subject in this task.

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Box 2: Decisions from Experience

272 There are several factors that may have contributed to the remarkable
273 performance of subjects in movement planning under risk [6-9, 44]. In these
274 experiments, the subject makes a long series of choices and over the course of the
275 experiment his/her accumulated winnings increase. In contrast, subjects in economic
276 decision-making experiments typically make a single “one-shot” choice, choosing among
277 a small set of lotteries. Indeed, when economic decision makers are faced with a series
278 of decisions they tend to move closer to maximum expected gain (see e.g. [45]; “the
279 house money effect.” [46]).

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Box 2's Figure 1 about here

283 Recent work suggests that subjects who are allowed to simulate a decision task
284 learn from their experience [47] and, together with the studies just cited, it is likely that
285 decision making improves with repetition. However, in the motor tasks discussed here
286 the learning phase does not involve explicit probabilities or values or tradeoffs between
287 risk and reward. In the experimental phase (Main text, Figure 2), they show no evidence
288 of learning. This outcome suggests that they can explicitly transfer experience with
289 motor uncertainty to the decision task (Figure 1), computing probabilities and planning
290 movements on demand. While subjects likely learn from experience in these motor
291 tasks, experience does not involve simple practice or simulation of the actual decision
292 task.

316 In detail, a movement strategy is a mapping from sensory input V to a
317 movement plan $S(V)$ (Figure 2). The expected gain associated with the choice of
318 strategy $S(V)$ is given by

$$319 \quad EG(s) = \iiint g(t, w) p_T(t | s(v)) p_V(v | w) p_W(w) dv dt dw \quad (3)$$

320 where W is the random state of the world (i.e., positions of arm, salt shaker, wine
321 glass, etc.) with prior distribution $p_W(w)$ based on past sensory information and
322 knowledge of how a table is laid out, V is current sensory information about the state of
323 the world with likelihood distribution $p_V(v | w)$ and T is the stochastic movement
324 trajectory resulting from the executed movement plan $S_T(V)$. The term $g(t, w)$
325 specifies the gain resulting from an actual trajectory t in the actual state of the world w .
326 In the example given it includes costs incurred by hitting objects while reaching through
327 the dinner scene and possible rewards for successfully grasping the salt shaker. Eq. 5
328 determines the movement strategy that maximizes expected gain.

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Box 3's Figure 2 about here

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Figure Legends

333 **Figure 1:** a) An example of a stimulus configuration presented on a display screen. The
334 subject must rapidly reach out and touch the screen. If the screen is hit within the green
335 circle, 2.5 cents is awarded. If within the red, there is a penalty of 12.5 cents. The circles
336 are small (9 mm radius) and the subject can't completely control this rapid movement. In
337 Box 1 we explain what the subject should do to maximize winnings. b) A comparison of
338 subjects' performance to the performance that would maximize expected gain. The shift
339 of subjects' mean movement end points from the center of the green target region is
340 plotted as a function of the shift of mean movement end point that would maximize
341 expected gain for 5 different subjects (indicated by 5 different symbols) and the 6
342 different target-penalty configurations shown in Fig. 2a (replotted from [6], Figure 5a).

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344 **Figure 2:** a) Trial-by-trial deviation of movement end point (in the horizontal direction)
345 from the mean movement end point in that condition as a function of trial number after
346 introduction of rewards and penalties (reward: 2.5 cents; penalty -12.5 cents); the six
347 different lines correspond to the six different spatial conditions of target and penalty
348 offset as shown on the right (data replotted from [6], Figure 7). b) Trend of a hypothetical
349 simple learning model in which a subject changes motor strategy gradually in response
350 to rewards and penalties incurred. The subject initially aims at the center of the green
351 circle. Before the subject's first trial in the decision-making phase of the experiment, the
352 subject is instructed that red circles carry penalties and green circles carry rewards.
353 Subjects may approach the aim point maximizing expected gain by slowly shifting the
354 aim point away from the center of the green circle until the winnings match the maximum
355 expected gain. However, the data shown in a) do not support this learning model.

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357 **Box 1's Figure 1:** Equivalence of a movement task and decision making under risk.
358 Subjects must touch a computer screen within a short period of time (e.g. 700 msec).
359 Subjects can win 2.5 cents points by hitting inside the green circle, lose 12.5 cents by
360 hitting inside the red circle, lose 10 cents by hitting where green and red circle overlap or
361 win nothing by hitting outside the stimulus configuration. Each possible aim point on the
362 computer screen corresponds to a lottery. a) Expected gain for a subject aiming at the
363 center of the green target (aim point indicated by the diamond). Black points indicate
364 simulated end points for a representative subject (with 5.6 mm end point standard
365 deviation); target and penalty circles have radii of 9 mm. This motor strategy yields an
366 expected loss of 2.8 cents/trial. The numbers shown below the target configuration
367 describe the lottery corresponding to this aim point, i.e., the probabilities for hitting inside
368 each region and the associated gain. b) Expected gain for a subject with the same motor
369 uncertainty as in a). Here, we simulate the same subject aiming towards the right of the
370 target center (the diamond) to avoid accidental hits inside the penalty circle. This
371 strategy results in an expected gain of 0.78 cents/trial and corresponds to the strategy
372 (aim point) maximizing expected gain. c) Each possible aim point corresponds to a
373 lottery and has a corresponding expected gain, shown by the grayscale background with
374 four particular aim points highlighted.

375

376 **Box 2's Figure 1: Motor decisions from experience.** In the learning phase of the
377 experiment subjects learn to hit targets. Their performance improves until their
378 movement variability has reached a plateau. During training they have the opportunity to
379 learn their own motor uncertainty but nothing about the training task requires that they
380 do so. In the experimental phase subjects plan movements that trade off the risk of
381 incurring penalties against the possible reward of hitting targets. They show little
382 evidence of learning and perform well in the task. This suggests that they can convert

383 what they learned in the training phase into the information needed to plan effective
384 movements under risk: the equivalent of estimating the probabilities of the various
385 outcomes associated with any proposed aim point, followed by a computation of
386 expected gain.

387

388 **Box 3's Figure 1:** Example of applying statistical decision theory to modeling goal-
389 directed movement under visual and motor uncertainty. a) A dinner guest, whose arm is
390 shown, intends to pick up the salt shaker at the center of the table with his right hand. An
391 intended trajectory is shown along with a "confidence interval" to indicate the range of
392 other trajectories that might occur. b) The actual executed movement may deviate from
393 the intended and, instead of grasping the salt shaker, the guest may accidentally knock
394 over his full wine glass. c) If executed successfully, the dinner guest will pick up the salt
395 shaker without experiencing social disaster. (Drawings by Andreas Olsson)

396

397 **Box 3's Figure 2:** Application of statistical decision theory to complex visuo-motor tasks.
398 The goal is a mapping from sensory input V to a movement plan $S(V)$. Gains and
399 losses $g(t, w)$ are determined by the actual trajectory t executed in the actual state of
400 the world w . The movement plan that maximizes expected gain depends on both visual
401 uncertainty and motor uncertainty. (Here, we follow the convention that random variables
402 are in upper case, e.g. X , while the corresponding specific values that those variables
403 can take on are in lower-case, e.g. $p(x)$.)

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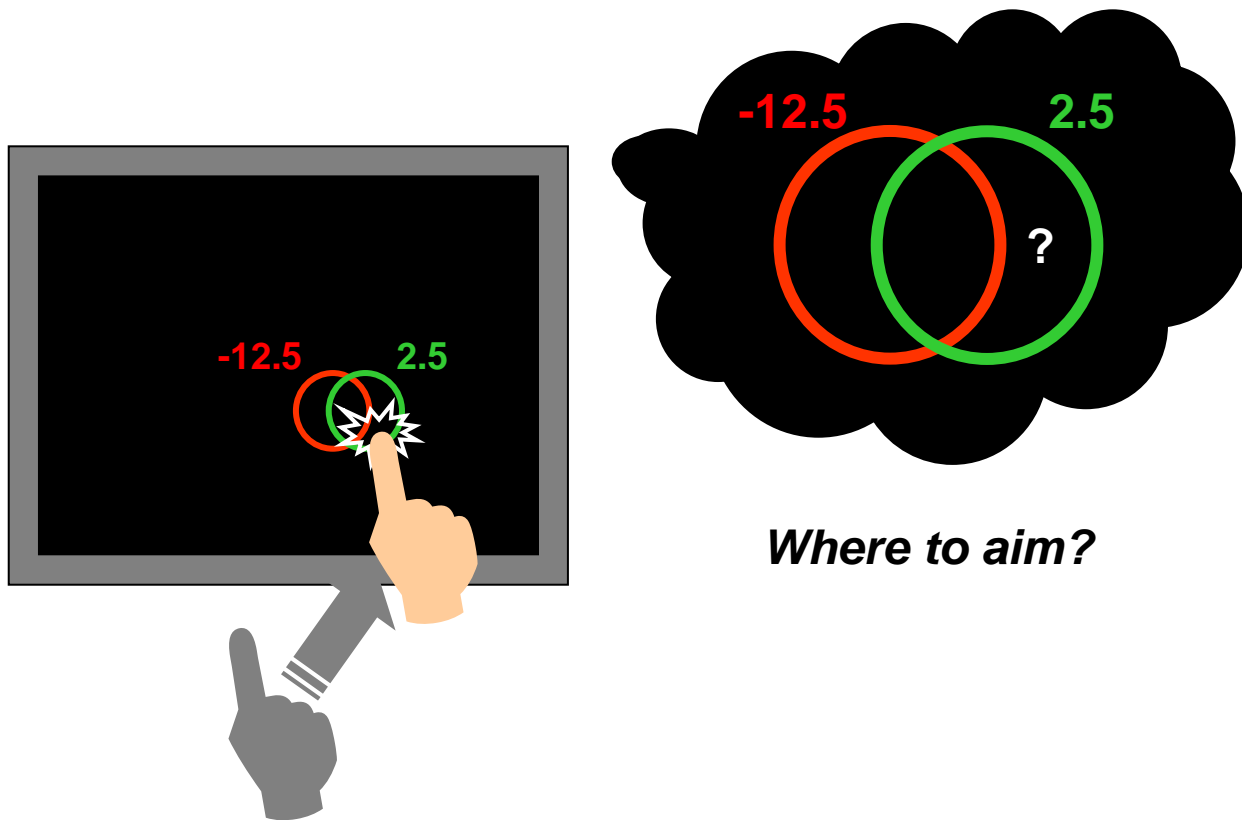
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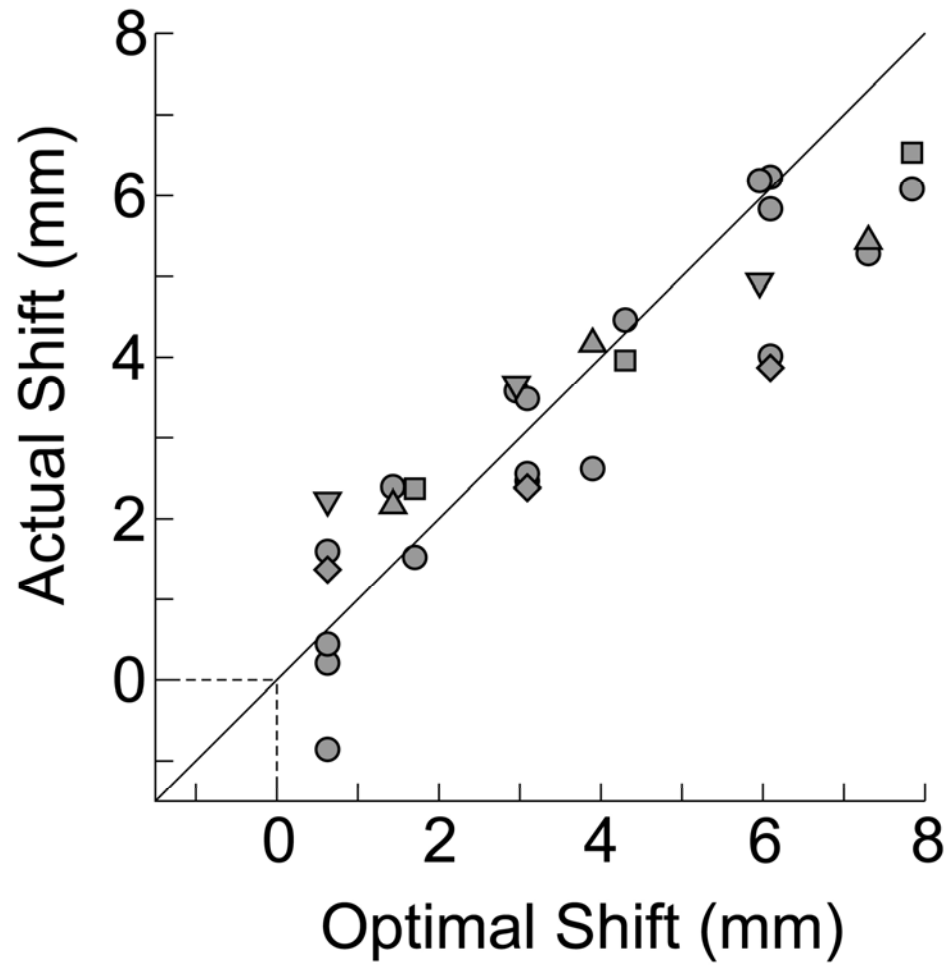
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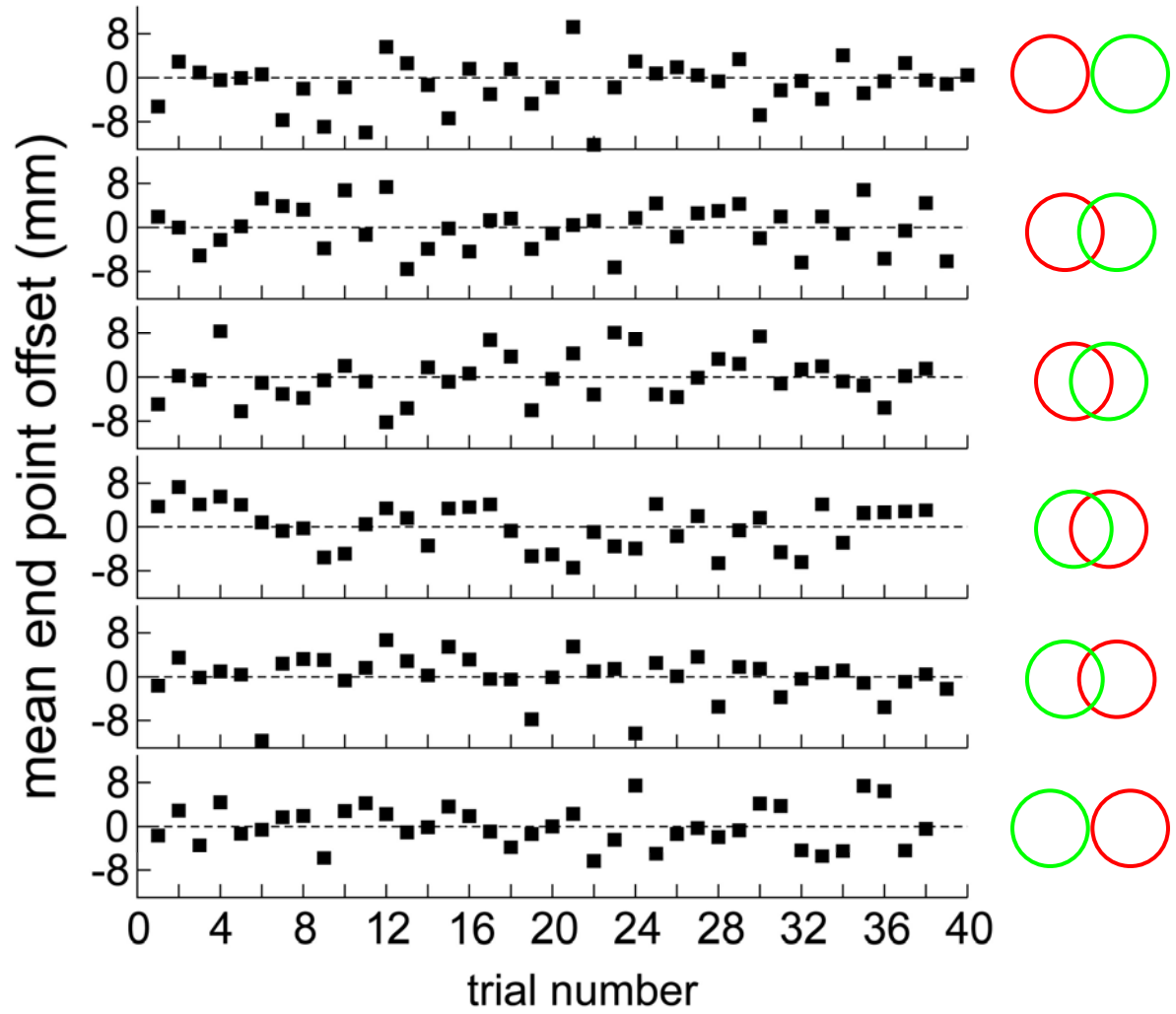
Main text, Figure 1a

C



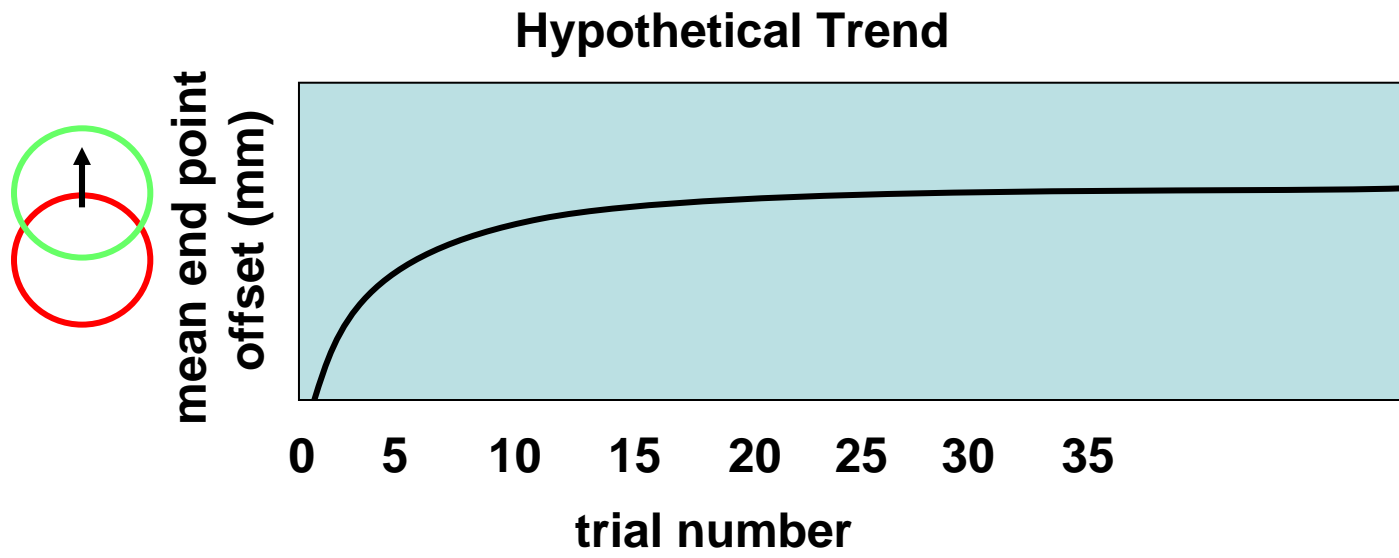
Main text, Figure 1b

a

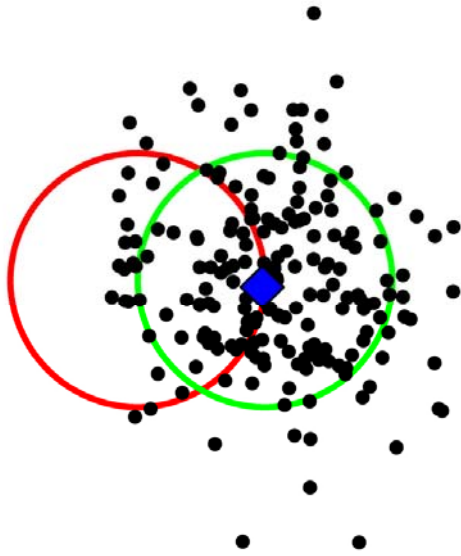


Main text, Figure 2a

b

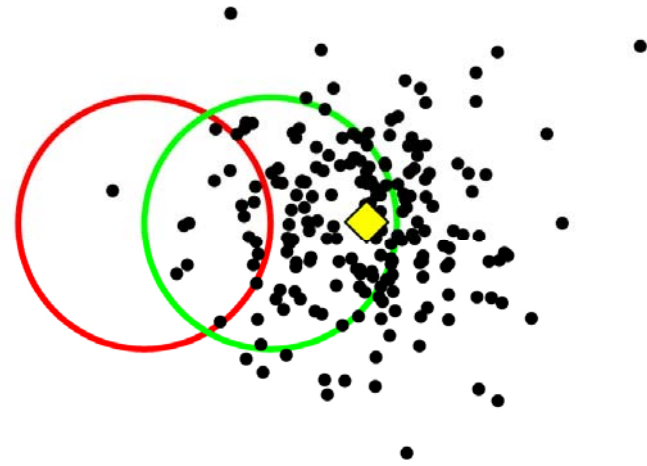


a



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0.29, -10 cents;
0.07, -12.5 cents;
0.21, 0 cents

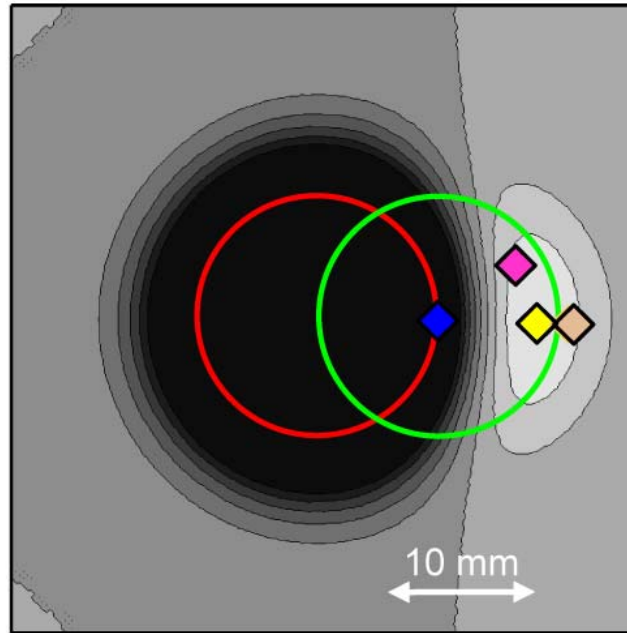
b



0.32, 2.5 cents;
0.03, -10 cents;
< 0.01, -12.5 cents;
0.65, 0 cents

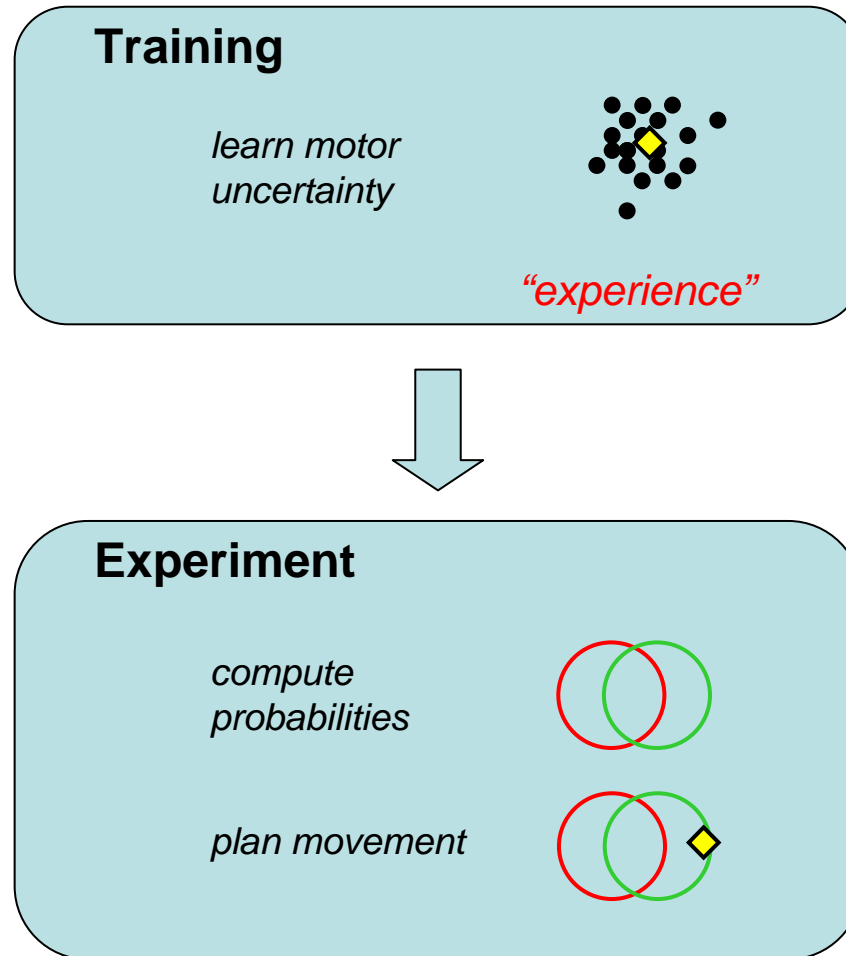
Box 1, Figure 1a and 1b

C



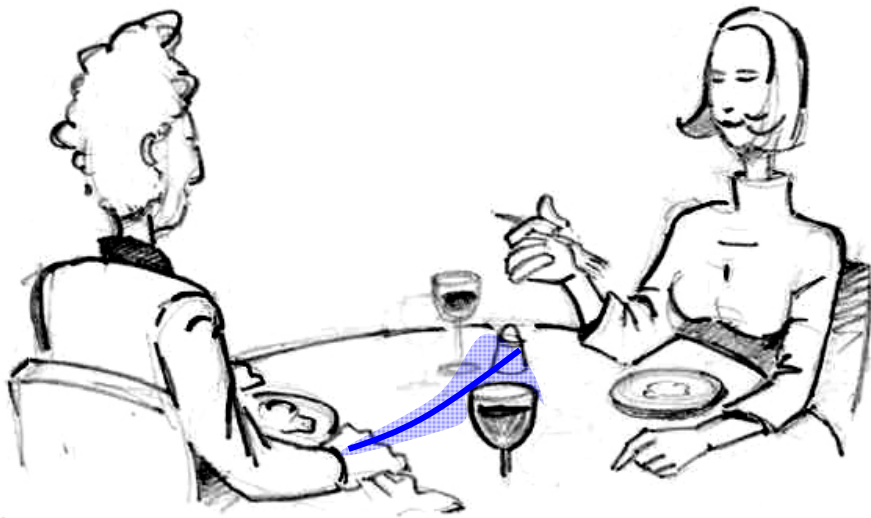
- ◆ 0.78 cents / trial
- ◆ 0.64 cents / trial
- ◆ 0.57 cents / trial
- ◆ -2.8 cents / trial

Box 1, Figure 1c



Box 2, Figure 1

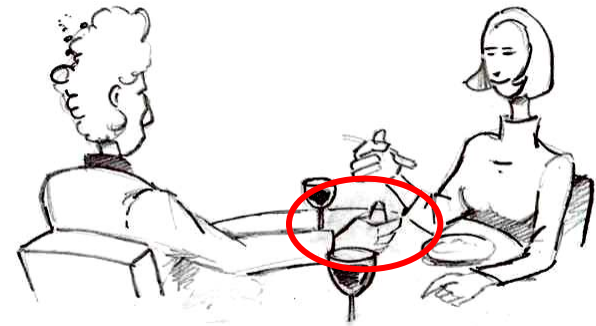
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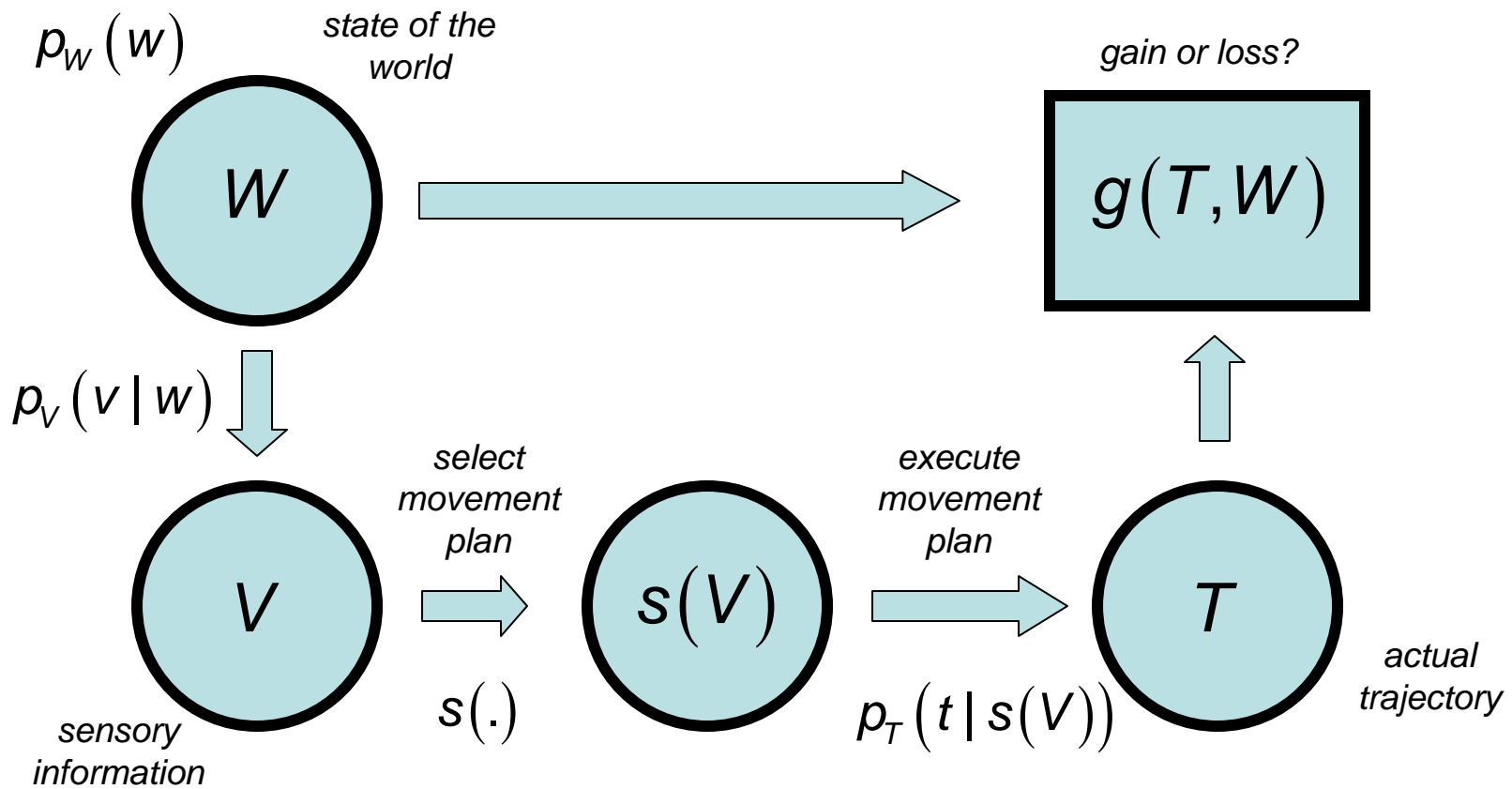
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c



Box 3, Figure 1



Box 3, Figure 2