The Scale Bandwidth of Visual Search

PREETI VERGHESE,*† DENIS G. PELLI*‡

Received 4 May 1993

Observers were asked to locate a target in a brief, two-scale display. Accuracy of locating the target was measured as a function of the ratio between the two scales. At each scale ratio, the probability of locating the target as a function of the number of elements is well fit by the idea that the observer accurately monitors only a “critical” number of elements. The dependence of critical number on scale ratio is well accounted for by a model that assumes that the observer’s decision is based on an evenly spaced array of samples. The sample spacing is under attentional control, but is always uniform.

INTRODUCTION

Can observers performing a visual search task simultaneously attend to information at a wide range of spatial scales? Sperling and Melchner (1978) investigated the ability of observers to search for targets in mixed-scale displays. Observers were presented with a block of small letters surrounded by a band of large letters. Two target numerals appeared, one among the small letters and one among the large letters. Each target numeral was always the same size as the distractor letters among which it appeared. The observer’s task was to identify and locate both target numerals. Sperling and Melchner showed that observer performance in such mixed-scale displays is worse than in single-scale displays. Our experiments are similar—measuring the effect of mixing scales on the ability to locate—but we use our “critical number” model to analyze the results.

The critical number model describes the probability of finding a target among many distracters as a function of the number of elements in a brief display (Verghese & Pelli, 1992). The critical number \( k \) is the number of elements in a brief display that a subject can monitor simultaneously. When the display has up to the critical number of elements, the probability of finding the target is 1. If the display has more than \( k \) elements, the probability is only \( k/n \), where \( n \) is the number of elements in the display. Verghese and Pelli showed that the critical number model fits data from four different attentive tasks. The data in Fig. 1 are from one such task in which the observer was asked to locate a static check in a flickering checkerboard, i.e. to “find the dead firefly”. The solid symbols plot the observer’s probability of finding the target check as a function of the total number of checks in the checkerboard. The critical number model’s fit is plotted as a solid line. The fit has a critical number \( k \) of 13, so the fitted probability is 1 up to 13 checks and then falls as \( 13/n \). Although the fit is imperfect (the data tend to round the corner, dropping below 1 before the best-fitting critical number is reached) the model still provides a fair fit and a useful one-parameter summary of our data in each experimental condition.

This task is “attentive” by both accuracy and reaction time criteria. Accuracy is inversely proportional to the number of checks (Fig. 1 above; Bergen & Julesz, 1983; Verghese & Pelli, 1992), and reaction time is proportional to the number of checks (19.5 msec per check, Verghese, 1990; Treisman & Gelade, 1980).

METHODS

Observers were presented with a modification of the many-frame version of Verghese and Pelli’s (1992) flickering checks “dead firefly” experiment. The observer’s task was to locate the single static check among many flickering checks. The stimulus was made up of two checkerboards that abutted at the center of the display. The geometry of these checkerboards was varied by changing check size and center-to-center spacing. We did four experiments, each using a different stimulus configuration: a single-scale display [Fig. 2(a)], a mixed-scale display [Fig. 2(b)], a mixed-spacing display [Fig. 2(c)], and a mixed-size display [Fig. 2(d)]. Note that
FIGURE 1. Probability of locating a single static check among many randomly flickering checks as a function of the number of checks in the checkerboard. The solid line is the fit of the critical number model to the data. Each data point is the average of a 100 trials and the vertical bar through the data point is the 95% confidence interval. The checks were arranged as a single-scale display [Fig. 2(a)] with a check size of 0.26".

Observers fixated a cross at the center of the screen and initiated each trial, which caused the fixation cross to be replaced by the stimulus. Each subsequent 15 msec frame showed a pair of checkerboards, one on each side of fixation. From frame to frame, each check was randomly reassigned a new polarity, light or dark, except for the target check, which remained static. Finally all checks stopped flickering, remaining frozen, and the observer used a mouse-controlled cursor to locate the target check, which had been the single static check among the randomly flickering checks. The observer was given auditory feedback indicating a right or wrong response and visual feedback indicating the location of the target check. The single target check appeared with equal probability on either side of the display. Observers were asked to fixate the center cross and pay equal attention to both sides of the display. The duration of the dynamic presentation was 180 msec (12 frames), which is shorter than the 200-400 msec required to make a shift of attention (Weichselgartner & Sperling, 1987).* Each block of 100 trials used only one kind of display (fixed check size and spacing on each side). An approximately equal number of trials in each block had displays made up of 2, 4, 8, 16, 32 and 64 checks, in random order.

*The 200-400 msec time to shift attention (Weichselgartner & Sperling, 1987) is much greater than the 30-50 msec processing time per element inferred from the slope of the line relating reaction time to number of elements (e.g. Treisman & Gelade, 1980). Presumably this is because the observer batch-processes many elements in each attentive glimpse, before shifting attention. We use the critical number model to measure the number of elements than an observer processes in each batch.

FIGURE 2. Stimulus configurations. (a) Single-scale display: the check size (and the center-to-center spacing) were identical on both sides of the display and ranged from 0.065" to 1.95". (b) Mixed-scale display: on the right side the check size (and spacing) were fixed at 0.26", and on the left they ranged from 0.065" to 1.95". (c) Mixed-spacing display: on the right side the check size and spacing were fixed at 0.26", and on the left the check size was also fixed at 0.26" while spacing ranged from 0.26" to 1.95". (Checks cannot overlap, so spacing cannot be less than size.) (d) Mixed-size display: on the right side the check size and spacing were fixed at 0.26", and on the left the spacing was also fixed at 0.26" while check size ranged from 0.065" to 0.26". (Again checks cannot overlap, so check size cannot be greater than spacing.)
order. Seven blocks of trials—totalling at least 100 trials per number-of-checks condition—were run for every combination of check size and spacing that we used in our experiments. Six check sizes were used, from 0.065°×0.065° to 1.95°×1.95°. This corresponds to a factor of 30 in linear size, or a factor of 900 in area. The luminances of the light and dark checks were 159 cd/m² and 25 cd/m², respectively; the checkerboards were surrounded by a gray background with a luminance of 78 cd/m². See Verghese and Pelli (1992) for other methodological details.

Two observers participated in the study. One of them was the first author; the other was unaware of the purpose of the experiment. The observers were 28–30 years old with normal acuity or acuity corrected to normal. The analysis has placed much more emphasis on the detection of the target in the left half than in the right half of the display, but the subjects (including the first author) had no inkling of this at the time of the experiments, and faithfully tried to divide their attention equally between the two sides.

RESULTS

Experiment 1: single-scale display

As a preliminary experiment, we used a single-scale display with the same check size and spacing on both sides of the display [Fig. 2(a)]. The probability of locating the target was measured as a function of the number of checks, which ranged from 2 to 64. This was done for six different check sizes ranging from 0.065°×0.065° to 1.95°×1.95°. The data for each check size were fit by the critical number model. The solid squares in Fig. 3 show critical number as a function of the check size for both observers. The critical number shown is for the left half of the display. (The horizontal scale is labeled “center-to-center spacing”, which in this case equals check size since the checks are contiguous. The spacings are relative to 0.26°.) This preliminary experiment provided the reference against which mixed-scale performance could be compared. In the rest of the experiments, the right-hand half of the display was fixed at the size, 0.26°, that produced the highest critical number.

Experiment 2: mixed-scale display

For the mixed-scale display [Fig. 2(b)] we varied the scale of the left half of the display while keeping a constant scale in the right half. For each scale ratio we measured the probability of locating the target as a function of the number of checks, and fit the critical number model to the results. The open squares in Fig. 3 show critical number for the left half of the display as a function of left:right scale ratios of 0.25 7.5. (Again the horizontal axis is labeled spacing, which equals size, and is expressed relative to the fixed 0.26° size of the checks on the right.) Note that the critical number for the mixed-scale display falls as the scale ratio deviates from 1. The left halves of the single- and mixed-scale displays are identical—only their right halves differ.

Figure 3 shows that shrinking or expanding the right half of a single-scale display to what, on its own, would be the optimal scale (0.26°), in fact makes performance on the left half worse, producing a tuning function that is narrower than that for a single scale.

Experiment 3: mixed-spacing display

We did two control experiments to disentangle the roles of size and spacing in the effects of mixing scales. Scale changes co-vary the size and center-to-center spacing of the checks. So, in this experiment, we kept check size constant, varying only the spacing of the checks on the left, as illustrated in Fig. 2(c). Figure 2(b) and (c) shows displays whose left sides have the same center-to-center spacing and the optimal scale (0.26°), in fact makes performance on the left half worse, producing a tuning function that is narrower than that for a single scale.
**Experiment 4: mixed-size display**

In this control experiment, we kept spacing constant and varied the size of the checks on the left side of the display, while the right side was fixed at 0.26', as illustrated in Fig. 2(d). We measured the critical number under these conditions for observer PV for left:right size ratios of 0.25 and 0.5. (Since the spacing was fixed and checks cannot overlap, the size ratio cannot exceed 1.) The symbols + and x in Fig. 3(b) represent the critical numbers for the left side of the display, for size ratios of 0.25 and 0.5 respectively, both plotted at a spacing ratio of 1. The data for these two size ratios nearly coincide with the filled square representing the single-scale condition, indicating that changing size, while keeping spacing constant, does not affect the critical number.

**DISCUSSION**

**Size or spacing?**

Changes in scale confound size and spacing, but the results of mixed-scale and mixed-spaced displays are not significantly different, indicating that nonuniform spacing accounts for the effect of nonuniform scale. Conversely, just mixing sizes has no effect. This was also demonstrated by Farell and Pelli (1993), who used a digit search task and a locating response, and found no effect of mixing characters of different sizes in a uniformly spaced array. All these results show that locating is impaired by inhomogeneity of spacing and unaffected by inhomogeneity of size.

**Uniform sampling: a model observer**

The fact that Expts 2 and 3 gave similar results showed that the effect of mixing scales is accounted for by the effect of mixing spacings. Changing the scale or spacing of a regular array changes its spatial frequency. It is inviting to imagine that the performance of this task is mediated by a single spatial-frequency channel, a la Campbell and Robson (1968).* In this spirit we wondered whether the observed variation of critical number with spacing ratio could be accounted for by assuming that the observer samples both sides of the display uniformly, with evenly spaced samples. The idea is that the observer's span of attention (or memory) can be described as a regular array of image samples (Nakayama, 1990; Van Essen, Olshausen, Anderson & Gallant, 1991). The spacing of these samples is under attentional control but is always uniform, and their number is given by the critical number for that spacing.

In order to account for our results, we defined a model observer. We assume that this observer may choose any uniform spacing of the samples; that the number of available samples depends on that spacing; and that the observer arranges the uniformly spaced and contiguous samples so as to maximize the number of checks monitored. The free parameters in the model are the spacing of the samples, the number of samples corresponding to that spacing, and the fraction of samples devoted to the left and the right. Since the conditions were blocked, we assume that the observer sets the spacing so as to maximize the probability of a correct response. We set the number of samples to be equal to the critical number measured in the single-scale experiment for this spacing, interpolating between measurements if necessary. Because we instructed observers to attend equally to the two sides of the display, in our modeling we initially assumed that the observer allocated half the samples to each side of our mixed displays. Thus the single-scale data fully determined the model, leaving no degrees of freedom for its prediction of the rest of the results.

In the model, each sample is the average luminance over a square area (though we take the artistic license of drawing it as circular in the figures). Adjacent samples are contiguous. At the end of the trial, we assume that the model observer computes each sample's variance over the many frames, and forgets the samples themselves. Recall that the target check is static, black or white; the rest of the checks are each randomly black or white on each frame. Therefore, the number of white checks in a sample not containing the target is a random sample from a binomial distribution \( b(m, 0.5) \), with a variance equal to \( m0.5^2 \), where \( m \) is the number of checks in the sample. If the sample contains the static target check then the variance is only \( (m - 1)0.5^2 \).

Among samples that cover an equal number of checks, the one with the least variance is the most likely to contain the target. Among samples that cover unequal numbers of checks we assume that the model chooses the sample with least variance, after normalization by the number of checks. When the chosen sample overlaps multiple checks then a check within that sample is chosen randomly as the designated "target" for purposes of scoring. We simulated the model's performance by running a thousand trials per number-of-checks condition for each spacing ratio. After obtaining the proportion correct as a function of number, we found the best-fitting critical number at each spacing ratio.

For a single-scale display the model observer places one sample on each check, sampling as many checks as the critical number at that scale. For a mixed scale display, the model uniformly samples the display at a single spacing, which results in redundant oversampling or ambiguous undersampling. Multiple samples of the same check are redundant. A sample that averages many checks is ambiguous.

**Oversampling**

Figure 4 shows 16 checks with a spacing ratio of 2:1. The superimposed circles represent the model observer's

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*The underlying idea here is that the observer monitors (or remembers) the display through a spatial-frequency bandpass filter, and that the area that the observer monitors corresponds to a fixed number of cycles at the frequency corresponding to the bandwidth of the filter. In this conceptualization the critical number is the number of samples specified by Nyquist's (1928) sampling theorem to adequately sample the filter output over the attended area: a space-bandwidth product.
samples. As noted above, for the mixed-scale display, the model observer adopts the spacing that maximizes performance, which in this case is the righthand spacing of 0.26°. This sample spacing results in there being one sample centered on each check on the right side, but many samples per check on the left side. Let \( A_{\text{left}} \) and \( A_{\text{right}} \) be the areas of unit cells i.e. squares with the same size as check spacing—on the left and right sides of the display. Let \( k_{0.26°} \) be the critical number measured on half a single-scale display with 0.26° checks. In this case the model observer takes \( k_{0.26°} \) samples on each side of the display. In Fig. 4, most of the samples on the left side are redundant, as there may be up to \( A_{\text{left}}/A_{\text{right}} \) samples per check, though only the first sample is informative. The critical numbers, \( k_{\text{left}} \) and \( k_{\text{right}} \), for the two halves of a mixed display can be estimated by computing how many checks are sampled on each side

\[
\begin{align*}
k_{\text{left}} &= \frac{A_{\text{left}}}{A_{\text{right}}} k_{0.26°} \\
k_{\text{right}} &= k_{0.26°}.
\end{align*}
\]

The lower bound of \( k_{\text{left}} \) corresponds to the case when each check on the left has the maximum number of samples, i.e., \( A_{\text{left}}/A_{\text{right}} \).

Suppose that the half-field critical number corresponding to the righthand spacing (0.26°) in Fig. 4 is 12. Then by equation (1) the model observer monitors \( k_{0.26°} = 12 \) checks on the right, and a minimum of \( A_{\text{right}}/A_{\text{left}} \times k_{0.26°} = 3 \) checks on the left. However, the model specifies that the contiguous evenly spaced samples should be arranged so as to maximize performance. By arranging the samples in the manner shown in Fig. 4, six large checks can be monitored on the left. The model’s predictions for this oversampling regime are compared with the data of our observers in the right halves of Fig. 6(a) and (b).

### Undersampling

Figure 5 shows 16 checks with a left:right spacing ratio of 0.5:1. The superimposed circles represent the model observer’s samples. The optimum model spacing is two-thirds the righthand spacing of 0.26°, but, for purposes of illustration, Fig. 5 shows a spacing of 0.26°, with \( k_{0.26°} \) contiguous samples on each side of fixation. There is one sample per check on the right side, but there is only one sample for every \( A_{\text{right}}/A_{\text{left}} \) checks on the left side. Figure 5 illustrates the case \( A_{\text{right}}/A_{\text{left}} = 4 \); each sample is the average of four checks. The model’s predictions for this undersampling regime are compared with the data of our observers in the left halves of Fig. 6(a) and (b).

### Data and predictions

Figure 6 plots the critical number for each side of the mixed displays as a function of spacing. In these displays, spacing on the left is varied while spacing on the right is fixed. Solid and open symbols plot the critical numbers for the right and left halves of the display, respectively. For both observers the critical numbers on the right are roughly constant, independent of the spacing on the left side. This suggests that the observer tended to sample the display at the righthand spacing and devoted half the samples to each side. It is natural for the observer to favor the righthand spacing since we set it to match the spacing that the observer performs best at in a single-scale display (Fig. 3). The solid and dashed lines in Fig. 6(a) and (b) are the model predictions.
predictions for the right and left halves of the display, respectively. The predicted critical number is based on
the sample spacing that maximized the total critical number over the whole display. For spacing ratios
greater than or equal to one, a sample spacing of 0.26°
turns out to be optimal. For spacing ratios less than one,
the optimal sample spacing is two-thirds the constant
righthand spacing of 0.26°. (Sampling at a spacing of
0.26° would perform much worse, especially at a spacing
ratio of 0.5:1.) Overall, the solid prediction is a good fit

FIGURE 7. Two possible sampling strategies for an observer sampling a mixed-scale display (with a scale ratio of 7.5:1) at
the smaller spacing. The half-field critical number corresponding to the righthand spacing is 11.5. (a) The observer devotes
this number of samples to each side. (b) The observer devotes more samples (16) to the left side, thereby increasing the number
of checks monitored on this side.
to the observed critical number on the right-hand side of
the display.

The dashed prediction, like the data, describes an
inverted V. Some data points deviate somewhat from the
predicted curve, but mostly there is good agreement,
affirming our supposition that the observers sampled
these mixed displays at a uniform spacing. Let us first
examine the deviations at spacing ratios greater than
one: oversampling. Observer PV's data deviate from the
oversampling prediction for the largest spacing ratios:
the righthand critical number is slightly less than pre-
dicted, and the lefthand critical number is higher than
predicted. Could these deviations represent a variation
of sampling strategy within the general constraints of
our sampling model? One might suppose that the ob-
server sampled the display at a spacing intermediate
between the two spacings in the display, sampling the
right side at a smaller spacing and the left side at a larger
spacing, but we discount this sampling strategy as the
resulting undersampling on the right side would severely
underestimate the observed critical number on that side.
Instead we suspect that the observer devoted more
samples to the side with the lower critical number,
despite the instruction to "divide attention equally". con-
sider observer PV's data for a scale ratio of 7.5. The
observed critical numbers for the small and large scale
are 7 and 4 respectively whereas the predicted values are
11.5 and 2. The prediction of these values is illustrated
in Fig. 7(a), assuming that observer PV devoted
11.5 = k_25 samples to each side. If, however, she
devoted 7 samples to the right side and 16 samples to the
left side, as shown in Fig. 7(b), then the observed critical
numbers of 7 and 4 would result, though, admittedly,
this is stretching things a bit. This strategy of devoting
more samples to the side with the lower critical number
is consistent with reports from observers that they try
to compensate for poor performance at the larger spac-
ing by attending to that side at the expense of the other
side.

Now consider the deviations at spacing ratios less than
one: undersampling. For a spacing ratio of 0.25:1, the
deviations from the prediction are small and in opposite
directions for the two observers, so the data at this
spacing ratio seem to be consistent with the model.

If our model observer were a perfect model of the
human observer then the target should only be seen by
the human at locations sampled by the model. Analysis
of our data gives a qualitative confirmation: the human
probability of detecting a target is much higher for
locations sampled by the model observer. For spacing
ratios of 2:1, 4:1 and 7.5:1, separately considering
checks on the left and right sides, the probability of
correctly locating a target at a model-sampled check is
high (in the range 77.1-84.4%), whereas the probability
at an unsampled check is low (15.3-31%). This goes in
the right direction, but, quantitatively, correction for
guessing and ambiguity of the target won't make these
numbers the 0 and 100% that our simple model predicts,
indicating that observers have some knowledge of the
checks not sampled by the model and an imperfect

FIGURE 8. Probability of a correct response in locating and identify-
ing tasks for a mixed-scale display with a spacing ratio of 2:1. There
are 16 checks on each side of fixation. The lightness of a check
represents the probability of a correct response at that location,
corrected for guessing, for observer PV. There are ten shades of
gray with the darkest representing a probability of 0.9-1.0, and the
lightest representing a probability of 0.0-0.1. The superimposed circles
are the samples of the scale-tuning model at the optimum spacing of
0.26°. (a) Data for the locating task, based on 100 trials per target
location, (b) data for the identifying task, based on 50 trials per target
location, and (c) the difference, the probability of identifying minus
the probability of locating. The + and − symbols indicate the sign of
the difference.
knowledge of the model-sampled checks, i.e. a less-than-full correspondence between the area monitored by the observer and the area sampled by the model.

Our model's contiguous sampling implies that the attended area is undivided. Figure 8(a) shows a "map of attention" for observer PV, showing her probability of accurately locating the target check at each site. The lightness of each check is proportional to the measured probability of locating the target at that site. The ten shades of gray represent the probability in steps of 0.1, with the darkest corresponding to a probability of 0.0-0.1 and the lightest corresponding to a probability of 0.9-1.0. The superimposed circles represent the model observer's samples. The lightest checks are contiguous and cover an area similar, though not identical to the area sampled by the model.

**Locating versus identifying**

Farell and Pelli (1993) studied attentive tasks like the one studied here, but they found an important difference in the results depending on whether they asked the observer to locate the target (as here) or to identify it. For various different attentive searches—numerals among letters as well as locating a static check in a flickering checkerboard—they consistently found that the locating response was worse in mixed- than in single-scale displays, while the identifying response was unimpaired. This is illustrated in Fig. 8(b), which shows the probability of a correct response at each site, after correction for guessing, when, instead of locating, observer PV was instead asked to identify the color (white or black) of the static check in a display identical to the one used in the locating task of Fig. 8(a). The probability of identifying the target is higher than the probability of locating it, especially at the more eccentric locations, which is more obvious in Fig. 8(c), which displays the difference between Fig. 8(a) and (b). Since the locating and identifying tasks ask the observer to search for the same target, one might have imagined that the two tasks would share the same search process, diverging only once the target was found and the observer had to encode either its identity or location. However, our model for locating posits that the locating performance is limited by the finite evenly spaced sampling array, whereas Farell and Pelli, and Fig. 8(b), indicate that no such limitation exists for identifying. This suggests that despite their similarity the two tasks may involve different search processes.

**CONCLUSION**

The results confirm that locating a target is harder in a mixed- than in a single-scale display (Sperling & Melchner, 1978; Farell & Pelli, 1993). Scale confounds size and spacing, but inhomogeneity of size has no effect, and inhomogeneity of spacing fully accounts for the effect of mixing scales. At each spacing ratio, the probability of locating the target as a function of number of elements is well fit by the idea that the observer accurately monitors only a "critical" number of elements. The dependence of critical number on scale ratio is well accounted for by a model that assumes that the observer's decision is based on an evenly spaced array of samples. The sample spacing is under attentional control, but is always uniform.

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Acknowledgements—This work was supported by NIH grant FY9442 to Denis Pelli. Preeti Verghese was supported by grant AFOSR-90-0330 to Ken Nakayama during the writing of this manuscript. Some of these results appeared in Preeti Verghese's Ph.D. thesis (Verghese, 1990). Katey Burns, Bart Farell and Josh Solomon provided helpful comments on the manuscript. We thank Mike Morgan for pointing out that it would be fruitful to separately analyze the data on both sides of the mixed-scale displays. We thank Murray Glanzer for asking whether the target was missed more by the human observer when it was outside the area sampled by the model observer. We thank Bobby Klatsky for suggesting the corner-rounning explanation given in the first footnote. And we thank Allen Potinson who wondered aloud why we hadn't done what is now Exp 4.